On the Channel Capacity of MIMO-Radar-Based Communications

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***Abstract*—Radar self-communication makes full use of the radar hardware resource and spectrum to fulfill the communication. As MIMO radar has become the new generation radar technology to be actively studied, MIMO radar self-communication and its capability are plausible to be investigated. The baseband models of MIMO radar and MIMO radar self-communication are set up, and radar mutual information is used to evaluate the radar performance. The optimal MIMO radar waveforms are derived to avoid the situation that the radar performance is degraded when the baseband waveform is modulated by the communication symbols. The channel capacity of radar self-communication is investigated under three situations, that is, without considering the radar performance, without channel state knowledge, and on condition that the optimal radar mutual information being guaranteed. Simulations show that the channel capacity of radar self-communication endures no loss in the high SNR region.**

I. Introduction

The notion of Radar-based communication was coined in the necessity of networking radars on one hand [1][2][3], and the deep exploration and utilization of radar spectrum due to spectrum scarcity on the other hand. To improve the capability of detecting and tracking, it is effective to fuse the data from several spatially distributed radars. Therefore, the communications between radars should be reliable and real-time. Meanwhile, it has been witnessed that radio frequency has turned into a seemingly diminishing commodity. It is difficult to integrate the new wireless application with a large amount of bandwidth requirement into the overly crowded radio spectrum, especially below 3GHz. Allowing spectrum sharing and coexistence among multiple systems can be a good solution.

In the past several decades, a great deal of research has been done in the design of radar-based communication systems. Without being equipped with a dedicated communication systems, radar shares the hardware, including antenna, RF module or more, with the communication function. This kind of integration is possible because of the analogy between two self-contained radio functionalities. The joint radar-communication system not only alleviates the problem of spectrum scarcity, but also reduces the volume and weight of the loader by reusing the hardware.

Lately, a great interest has emerged in MIMO radars, which have parallels with multiple-input multiple-output (MIMO) systems in wireless communications. According to the configuration of the antenna array, MIMO radars can be divided into two categories. One is widely-separated MIMO radar [4], the inter-element spacing of which is much larger than the wavelength. Illuminating the targets from different angles, widely-separated MIMO radar makes full use of space diversity to reduce the radar cross-section (RCS) fluctuations. The other is collocated MIMO radar [5]. The inter-element spacing is equal to half wavelength or comparable with it. No matter which MIMO radar system is concerned, we assume that multiple transmitting antennas are employed to emit different waveforms and multiple receiving antennas process the echoes. Additionally, the distance between the transmitter/receiver elements is greater than or equal to half wavelength.

MIMO radar is also expected to implement communication. Researchers have begun to address the practical issues related to this novel communication paradigm. As Donnet and Longstaff have proposed [6], orthogonal frequency division multiplexing (OFDM) can be applied to MIMO radar to achieve both radar and communication capability. In the candidate system, the whole band is divided into many groups of subcarriers with each group corresponding to an antenna element. In each subcarrier group, one subcarrier is for pilot and the others are used for data symbol. To ensure robust operation for both radar and communication functions, each data symbol is spread by pulse compression code.

In this paper, we do not consider the practical details of the MIMO radar-based communication system implementation, for there is a more interesting topic on how much capacity this novel communications system can achieve without degrading the radar's original capability. Although used for different purposes, MIMO radar and MIMO communication have many similarities, especially in the information derivation. Mutual information has been applied in MIMO radar waveform design [7], and yet mutual information has been more intensively applied to MIMO channel capacity analysis. Therefore, information theory can bridge the MIMO radar with the MIMO radar-based communications. We use the mutual information to evaluate the capacity of MIMO radar based communication and the mutual influence between radar and communication.

The outline of the paper is as follows. Section III introduces the baseband signal model of MIMO radar and MIMO radar self-communication. Then, the optimal waveform design rule is derived in Section IV. In Section V, the channel capacity of MIMO radar self-communication under three conditions is discussed. Simulations are given in Section VI, followed by

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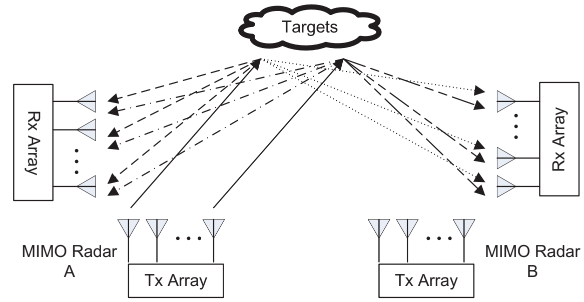

Fig. 1. Application scenario of MIMO-radar based communication.

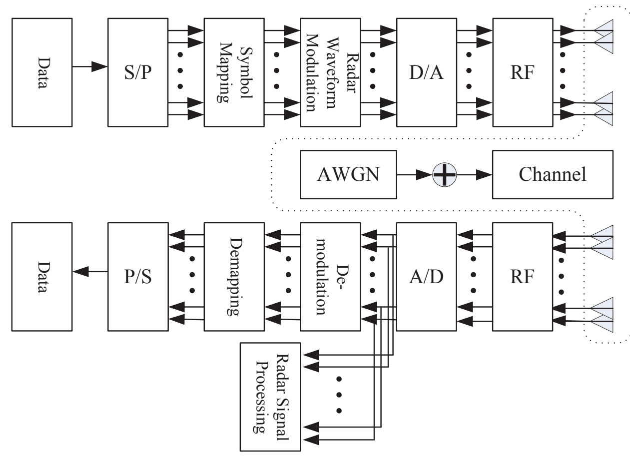

Fig. 2. The block diagram of MIMO radar with communication capability.

the concluding remarks in Section VII.

II. Our Contributions and Related Works

The key contributions of the paper and the related works are as follows.

- A generalized model of MIMO-radar-based communication system is given. Early studies on MIMO-radar-based communications, such as [1] and [6], mainly focused on subcarrier allocation at the transmitter and signal processing at the receiver [8]. The whole system modeling is rarely considered.
- The mutual information is employed to make a connection between MIMO radar and MIMO-radar-based communications. Yang and Blum [9] introduced the mutual information criterion into the waveform design of MIMO radar. Other existing works, such as [7], extended the work in [9]. At the same time, there is considerable amount of research on the capacity of typical MIMO communication channel, such as [10] and its references. The scenarios they discussed is different to that of this paper.
- The interaction between MIMO radar and MIMO-radar-based communications are also analyzed. Although there is a lot of effort to be spent on developing radar-based communication system, as [3] listed, little research has been conducted in applying mutual information to quantify the mutual influence between two functionalities.

III. System Model

The application scenario we study in this paper is shown in Fig. 1. Two MIMO radars cooperate to work. MIMO radar A uses multiple antennas to transmit uncorrelated signals, which are modulated by the data symbols. The dual-use signals are reflected by the scatters. Then the receiving antennas of MIMO radar A collect the echoes. At the same time, the signals of MIMO radar A arrive at the receiving ends of MIMO radar B, and the information carried is extracted by the communication module. Meanwhile, if two or more radars transmit simultaneously, their signal can be differentiated by the multi-access technique, such as frequency division multi-access (FDMA).

Assume that the far-field target is presented in the illuminating area, and there are N_T transmitting antennas and N_R receiving antennas of MIMO radar. The discrete-time baseband signal model of MIMO radar system is considered. Fig. 2 presents the block diagram of joint MIMO radar and communication system. The baseband data are mapped on to a symbol constellation, such as PSK (phase shift key), and the symbols are assigned to the antennas. After radar pulse shaping and D/A conversion, the symbols are converted to analog signal, which is sent to up-converter and power amplifier then to be transmitted.

Assuming that there are N_S symbols during a pulse with $N_S > N_T$ and $N_S > N_R$. The mth baseband symbol on the ith transmitting antenna is $S_i[m]$. Denote the baseband signal of the antenna array on the mth snapshot as a $N_T \times 1$ vector, $\mathbf{s}[m] = [s_1[m] \cdots s_i[m] \cdots s_{N_T}[m]]^T, \quad i = 1, \cdots, N_T$ with $[\cdot]^T$ being matrix transpose.

A. Signal Model at MIMO Radar Receiver

At the receiver of MIMO radar A, the received signal vector of the jth antenna can be written as

$$\mathbf{r}_j = \begin{bmatrix} r_j[0] & r_j[1] & \cdots & r_j[N_S - 1] \end{bmatrix}^T \tag{1}$$

and the mth sample is

$$r_j[m] = \sum_{i=1}^{N_T} g_{i,j} s_i[m] + z_j[m] \tag{2}$$

where $g_{i,j}$ is the pulse response between the ith transmitting antenna and the jth receiving antenna; $z_j[m]$ is the clutter received by the jth receiving antenna, including the noise and interference [11]. Assume that the clutter is subject to complex Gaussian distribution with zero mean and variance σ_n^2.

The echo signal can be also written in matrix-vector form as

$$\mathbf{r}_j = \mathbf{S}^T \mathbf{g}_j + \mathbf{z}_j \tag{3}$$

where $\mathbf{g}_j = \left[g_{1,j} \cdots g_{N_T,j}\right]^T$, $\mathbf{z}_j = \left[z_j[0] \cdots z_j[N_S - 1]\right]^T$, and $\mathbf{S} = [\mathbf{s}[0] \cdots \mathbf{s}[N_S - 1]]$.

Collect the receiving echo of all receiving antennas during a pulse, and construct received echo matrix Υ with $\{\Upsilon\}_{m,j} = r_j[m]$. Define the pulse response matrix as $\mathbf{G}$ with $\{\mathbf{G}\}_{i,j} = g_{i,j}$ and the clutter matrix $\mathbf{Z}$ with $\{\mathbf{Z}\}_{m,j} = z_j[m]$. Then the received echo is given as

$$\Upsilon = \mathbf{S}^T\mathbf{G} + \mathbf{Z} \tag{4}$$

Assume that the clutter matrix $\mathbf{Z}$ is uncorrelated with the transmitted baseband signal $\mathbf{S}$ and the impulse response matrix $\mathbf{G}$. We also assume that the spatial correlation matrix

$$\Sigma_G = E\left\{\mathbf{G}\mathbf{G}^H\right\}$$

is with full rank, where $E\{\cdot\}$ represents the expectation and $[\cdot]^H$ is the conjugate transpose of a matrix.

B. Signal of MIMO Radar Self-Communication System

As far as the MIMO radar communication is concerned, MIMO radar B extracts the communication information from the received signal, which experiences the different paths from the received signal arriving at the receiving end of MIMO radar A. The received signal of MIMO radar B is modeled as

$$\mathbf{Y} = \mathbf{H}\mathbf{S} + \mathbf{N} \tag{5}$$

where $\mathbf{Y} = [\mathbf{y}[0]\cdots\mathbf{y}[m]\cdots\mathbf{y}[N_S - 1]]$ and the mth column is $\mathbf{y}[m] = \left[y_1[m]\cdots y_j[m]\cdots y_{N_R}[m]\right]^T$; $\mathbf{S}$ is the transmitted baseband vector and defined the same as the above; $\mathbf{N}$ is the circularly symmetric complex white Gaussian noise vector, with zero mean and vairance $\sigma_n^2\mathbf{I}_{N_T}$, which has the same statistics as the clutter of MIMO radar A. The channel transfer matrix $\mathbf{H}$ is invariant during a pulse and $\mathbf{h}_i$ denotes its ith column. Therefore, the received signal vector at the mth sample moment is written as

$$\mathbf{y}[m] = \mathbf{H}\mathbf{s}[m] + \mathbf{n}[m] \tag{6}$$

where $\mathbf{n}[m]$ denotes the mth column of $\mathbf{N}$.

IV. Waveform Design of MIMO Radar

The mutual information between target impulse response $\mathbf{G}$ and echo signal Υ can be used to measure the radar performance [9][12]. On the condition that MIMO Radar A has known the transmitted baseband signal $\mathbf{S}$, which is radar baseband waveform as well as self-communication signal, the mutual information between $\mathbf{G}$ and Υ is calculated as

$$I(\mathbf{G};\Upsilon|\mathbf{S}) = h(\Upsilon|\mathbf{S}) - h(\Upsilon|\mathbf{G},\mathbf{S}) = h(\Upsilon|\mathbf{S}) - h(\mathbf{Z}) \tag{7}$$

where $h(\cdot)$ denotes the entropy. Since $\mathbf{r}_j$'s are assumed to be independent, the conditional probability density function (pdf) of Υ for a given $\mathbf{S}$ is

$$\begin{aligned} p(\Upsilon|\mathbf{S}) &= \prod_{j=1}^{N_R} p\left(\mathbf{r}_j\middle|\mathbf{S}\right) \\ &= \prod_{j=1}^{N_R} \frac{1}{\pi^{N_S}\det\left(\mathbf{S}^T\Sigma_G\mathbf{S}^* + \sigma_n^2\mathbf{I}_{N_S}\right)} \times \\ &\quad \exp\left(-\mathbf{r}_j^H\left(\mathbf{S}^T\Sigma_G\mathbf{S}^* + \sigma_n^2\mathbf{I}_{N_S}\right)^{-1}\mathbf{r}_j\right) \\ &= \frac{1}{\pi^{N_R N_S}\det^{N_R}\left(\mathbf{S}^T\Sigma_G\mathbf{S}^* + \sigma_n^2\mathbf{I}_{N_S}\right)} \times \\ &\quad \exp\left(-tr\left(\left(\mathbf{S}^T\Sigma_G\mathbf{S}^* + \sigma_n^2\mathbf{I}_{N_S}\right)^{-1}\Upsilon\Upsilon^H\right)\right) \end{aligned} \tag{8}$$

where $\det(\cdot)$ and $tr(\cdot)$ obtain the determinant and trace of a matrix and $E\left\{\mathbf{Z}\mathbf{Z}^H\right\} = \sigma_n^2\mathbf{I}_{N_S}$. The conditional differential entropy is calculated as

$$h(\Upsilon|\mathbf{S}) = N_R N_S \log(\pi) + N_R N_S + N_R \log\left[\det\left(\mathbf{S}^T\Sigma_G\mathbf{S}^* + \sigma_n^2\mathbf{I}_{N_S}\right)\right] \tag{9}$$

By the same token, the clutter matrix $\mathbf{Z}$ has the multivariate complex Gaussian distribution and its pdf is written as

$$p(\mathbf{Z}) = \frac{1}{\pi^{N_R N_S}\left(\sigma_n^2\right)^{N_R}}\exp\left(-tr\left(\left(\sigma_n^2\mathbf{I}_{N_S}\right)^{-1}\mathbf{Z}\mathbf{Z}^H\right)\right) \tag{10}$$

And its entropy is

$$\begin{aligned} h(\mathbf{Z}) &= -\int p(\mathbf{Z})\log p(\mathbf{Z})d\mathbf{Z} \\ &= N_R N_S \log(\pi) + N_R N_S + N_R \log\left[\det\left(\sigma_n^2\mathbf{I}_{N_S}\right)\right] \end{aligned} \tag{11}$$

Substitute (11) and (9) into (7), the mutual information is calculated as

$$I(\mathbf{G};\Upsilon|\mathbf{S}) = N_R \log\left[\det\left(\sigma_n^{-2}\mathbf{S}^T\Sigma_G\mathbf{S}^* + \mathbf{I}_{N_S}\right)\right] \tag{12}$$

A. Optimal Power Assignment of MIMO Radar Waveform

The self communication capability is the auxiliary function of radar system, and its design principle is to minimize the negative influence with radar performance. So it is obliged to guarantee the maximum of the radar mutual information at first, and then design the communication signal in order to obtain the channel capacity with the largest probability.

In order to achieve the maximum radar mutual information, the transmitting waveform $\mathbf{S}$ should be designed according to target impulse response $\mathbf{G}$ to make received signal Υ containing more information about $\mathbf{G}$. The spatial correlation matrix can be diagonalize through eigenvalue decomposition, i.e.

$$\Sigma_G = E\left\{\mathbf{G}\mathbf{G}^H\right\} = \mathbf{U}_G\mathbf{\Lambda}_G\mathbf{U}_G^H \tag{13}$$

where $\mathbf{U}_G$ is a unitary matrix and $\mathbf{\Lambda}_G = diag\left\{\lambda_{11}, \cdots \lambda_{ii}, \cdots \lambda_{N_T N_T}\right\}$ is diagonal matrix with λ_{ii} being the eigenvalue. Recall that

$$\det\left(\mathbf{A}_{N_R\times N_T}\mathbf{B}_{N_T\times N_R} + \mathbf{I}_{N_R}\right) = \det\left(\mathbf{B}_{N_T\times N_R}\mathbf{A}_{N_R\times N_T} + \mathbf{I}_{N_T}\right) \tag{14}$$

Eq. (7) can also be written as

$$I(\mathbf{G};\Upsilon|\mathbf{S}) = N_R \log\left[\det\left(\sigma_n^{-2}\mathbf{\Lambda}_G\left(\mathbf{S}^T\mathbf{U}_G\right)^H\mathbf{S}^T\mathbf{U}_G + \mathbf{I}_{N_T}\right)\right] \tag{15}$$

Define $\mathbf{Q} = \left(\mathbf{S}^T\mathbf{U}_G\right)^H \mathbf{S}^T\mathbf{U}_G$, and its (i, j)th entry is q_{ij}.

There exists the Hadamards inequalities for the determinant and trace of an $N_T \times N_T$ positive semi-definite Hermitian matrix

$$\det\left(\mathbf{Q}_{N_T\times N_T}\right) \le \prod_{i=1}^{N_T} q_{ii} \tag{16}$$

and

$$tr\left(\mathbf{Q}^{-1}_{N_T\times N_T}\right) \ge \prod_{i=1}^{N_T} \frac{1}{q_{ii}} \tag{17}$$

and equalities are achieved in both cases if and only if $\mathbf{Q}_{N_T\times N_T}$ is diagonal. Thus we derive that

$$I\left(\mathbf{G}; \Upsilon | \mathbf{S}\right) \le N_R \log\left[\prod_{i=1}^{N_T}\left(\sigma_n^{-2}\lambda_{ii}q_{ii} + 1\right)\right] \tag{18}$$

Since $\mathbf{U}_G$ is unitary, it is easy to show that $tr(\mathbf{Q}) = tr\left(\left(\mathbf{S}^T\mathbf{U}_G\right)^H \mathbf{S}^T\mathbf{U}_G\right) = tr\left(\mathbf{S}^*\mathbf{S}^T\right)$ Note that the total power is limited. Now, we are in a position to allocate the power optimally on the transmitting antennas to obtain the maximum mutual information by solving the following problem,

$$\max_{\mathbf{Q}} \left\{\log\left[\prod_{i=1}^{N_T}\left(\sigma_n^{-2}\lambda_{ii}q_{ii} + 1\right)\right]\right\} \tag{19}$$
$$s.t. \qquad tr(\mathbf{Q}) \le N_T P, \qquad q_{ii} > 0, \quad 1 \le i \le N_T$$

where $N_T P$ refers to the total power. The Lagrange multipliers method is employed and a Lagrangian function is introduced as

$$L = \sum_{i=1}^{N_T} \log\left(\sigma_n^{-2}\lambda_{ii}q_{ii} + 1\right) + \gamma \sum_{i=1}^{N_T} q_{ii} \tag{20}$$

Differentiating L with respect to q_{ii} and setting it to 0, we obtain

$$\frac{q_{ii}}{\lambda_{ii}q_{ii} + \sigma_n^2} + \gamma = 0 \tag{21}$$

the optimality conditions are satisfied if

$$\sum_{i=1}^{N_T}\left(-\frac{1}{\gamma} - \frac{\sigma_n^2}{\lambda_{ii}}\right)^+ = N_T P \tag{22}$$

and

$$q_{ii} = \left(-\frac{1}{\gamma} - \frac{\sigma_n^2}{\lambda_{ii}}\right)^+ \quad 1 \le i \le N_T \tag{23}$$

Since the diagonal elements of $\mathbf{Q}$ are real and larger than 0, $\mathbf{Q}^{1/2}$ exists. For any $N_S \times N_T$ matrix $\mathbf{\Psi}$, if $\mathbf{\Psi}^H\mathbf{\Psi} = \mathbf{I}_{N_T}$, and set

$$\mathbf{S} = \mathbf{U}_G^* \mathbf{diag}\left\{ \sqrt{q_{11}}, \quad \cdots \quad \sqrt{q_{ii}}, \quad \cdots \quad \sqrt{q_{N_T}} \right\}\mathbf{\Psi}^T \tag{24}$$

we have

$$\mathbf{S}^*\mathbf{S}^T = \mathbf{U}_G\mathbf{Q}^{1/2}\mathbf{\Psi}^H\left(\mathbf{U}_G^*\mathbf{Q}^{1/2}\mathbf{\Psi}^T\right)^T = \mathbf{U}_G^*\mathbf{Q}\mathbf{U}_G^H \tag{25}$$

What has been discussed above is based on the hypothesis that the MIMO radar transmitter and receiver have known the transmitted baseband signal. Once $\mathbf{\Psi}$ is determined, so does $\mathbf{S}$. But as far as the radar self-communication is concerned, transmitted baseband signal $\mathbf{S}$ contains the information and is a random matrix. If $\mathbf{S}$ does not follow the optimal power allocation method derived above, the mutual information of radar has to bear loss. In the next section, the design of communication signal matrix is based on the optimal design of radar mutual information.

V. Channel Capacity of MIMO Radar Communication

The capacity of the MIMO communication channel is defined as the maximum mutual information between the transmitted signal and the received signal. For a sample realization of channel $\mathbf{H}$, the conditional pdf of $\mathbf{Y}$ is given as

$$\begin{aligned} p\left(\mathbf{Y}|\mathbf{H}\right) &= \prod_{j=1}^{N_S} p\left(\mathbf{y}[m]|\mathbf{H}\right) \qquad (26)\\ &= \prod_{j=1}^{N_S} \frac{1}{\pi^{N_R}\det\left(\mathbf{H}\Sigma_S\mathbf{H}^H + \sigma_n^2\mathbf{I}_{N_R}\right)} \times \\ &\quad \exp\left(-\mathbf{y}[m]^H\left(\mathbf{H}\Sigma_S\mathbf{H}^H + \sigma_n^2\mathbf{I}_{N_R}\right)^{-1}\mathbf{y}[m]\right) \\ &= \frac{1}{\pi^{N_S N_R}\det^{N_S}\left(\mathbf{H}\Sigma_S\mathbf{H}^H + \sigma_n^2\mathbf{I}_{N_R}\right)} \times \\ &\quad \exp\left(-tr\left(\left(\mathbf{H}\Sigma_S\mathbf{H}^H + \sigma_n^2\mathbf{I}_{N_R}\right)^{-1}\mathbf{Y}\mathbf{Y}^H\right)\right) \end{aligned}$$

with assumption that the columns of $\mathbf{S}$ are identical independently distributed (i.i.d) and Gaussian distributed, and $\Sigma_S = E\left\{\mathbf{S}\mathbf{S}^H\right\}$. Then the conditional entropy is written as

$$\begin{aligned} h\left(\mathbf{Y}|\mathbf{H}\right) &= -\int p\left(\mathbf{Y}|\mathbf{H}\right)\log p\left(\mathbf{Y}|\mathbf{H}\right)d\mathbf{Y} \\ &= N_R N_S \log(\pi) + N_R N_S + \\ &\quad N_S \log\left[\det\left(\mathbf{H}\Sigma_S\mathbf{H}^H + \sigma_n^2\mathbf{I}_{N_R}\right)\right] \end{aligned}$$

At the same time, the noise matrix $\mathbf{N}$ has the pdf as

$$p(\mathbf{N}) = \frac{1}{\pi^{N_R N_S}\left(\sigma_n^2\right)^{N_S}} \exp\left(-tr\left(\left(\sigma_n^2\mathbf{I}_{N_R}\right)^{-1}\mathbf{N}\mathbf{N}^H\right)\right) \tag{27}$$

and the entropy as

$$h(\mathbf{N}) = N_R N_S \log(\pi) + N_R N_S + N_S \log\left[\det\left(\sigma_n^2\mathbf{I}_{N_R}\right)\right]$$

By a simple calculation, we can easily obtain the mutual information between $\mathbf{S}$ and $\mathbf{Y}$ as

$$I(\mathbf{S}; \mathbf{Y}) = N_S \log\left[\det\left(\sigma_n^{-2}\mathbf{H}\Sigma_S\mathbf{H}^H + \mathbf{I}_{N_R}\right)\right] \tag{28}$$

Notice the difference between the two mutual information. For the MIMO radar, the receiver of the radar knows the transmitted baseband signal, but the target impulse response is random. While for MIMO radar communication, a realization of the channel is given, but the transmitted baseband signal is unknown. In the radar signal model, all the sampling points during a pulse are accumulated, while in the communication signal model, each sampling point is assumed to contain a communication symbol.

A. Communication Optimal without Considering Radar

Define $\mathbf{R_H} = \mathbf{H}^H\mathbf{H}$, which is a $N_T \times N_T$ positive semi-definite Hermitian matrix. Its eigenvalue decomposition is expressed as $\mathbf{R_H} = \mathbf{U_H^H \Lambda_H U_H}$, where $\mathbf{U_H}$ is a $N_T \times N_T$ unitary matrix; $\mathbf{\Lambda_H} = diag\{\mu_{11}, \cdots \mu_{ii}, \cdots \mu_{N_T N_T}\}$ is a diagonal matrix with the eigenvalues being the diagonal elements. Therefore,

$$\begin{aligned} \det\left(\sigma_n^{-2}\mathbf{H}\Sigma_S\mathbf{H}^H + \mathbf{I}_{N_R}\right) &= \det\left(\sigma_n^{-2}\mathbf{H}^H\mathbf{H}\Sigma_S + \mathbf{I}_{N_T}\right) \quad (29) \\ &= \det\left(\sigma_n^{-2}\mathbf{\Lambda_H U_H}\Sigma_S\mathbf{U_H^H} + \mathbf{I}_{N_T}\right) \end{aligned}$$

If $\mathbf{U_H}\Sigma_S\mathbf{U_H^H} = \mathbf{D}$ is a diagonal matrix, (28) achieves its maximum, where

$$\mathbf{D} = diag\left\{ \begin{array}{ccccc} d_1, & \cdots & d_i, & \cdots & d_{N_T} \end{array} \right\} \quad (30)$$

Since $tr\left(\mathbf{U_H}\Sigma_S\mathbf{U_H^H}\right) = tr\left(\Sigma_S\mathbf{U_H^H U_H}\right) = tr(\Sigma_S) = tr(\mathbf{D})$, the mutual information maximization problem now becomes

$$\begin{aligned} C = \max_{\mathbf{D}} &\left\{N_S \log\left[\det\left(\sigma_n^{-2}\mathbf{\Lambda_H^D} + \mathbf{I}_{N_T}\right)\right]\right\} \quad (31) \\ s.t. \quad & tr(\mathbf{D}) \le N_T P \\ & d_{ii} > 0, \quad 1 \le i \le N_T \end{aligned}$$

The problem will be solved in the same way as employed in the last section. The capacity is written as

$$C = N_S \log\left[\prod_{i=1}^{N_T}\left(\sigma_n^{-2}\mu_{ii}d_{ii} + 1\right)\right] \quad (32)$$

where

$$d_{ii} = \left(-\frac{1}{\beta} - \frac{\sigma_n^2}{\mu_{ii}}\right)^+ \quad 1 \le i \le N_T \quad (33)$$

and the Lagrangian multiplier β satisfies

$$\sum_{i=1}^{N_T}\left(-\frac{1}{\beta} - \frac{\sigma_n^2}{d_{ii}}\right)^+ = N_T P \quad (34)$$

Now, we consider a communication-radar system which firstly guarantees the performance of communication and then takes account of the radar capability. On the condition of achieving the channel capacity, this communication-radar system can get the radar mutual information at the same time as

$$\begin{aligned} I(\mathbf{G}; \Upsilon|\mathbf{S}) &= N_R \log\left[\det\left(\sigma_n^{-2}\Sigma_G\mathbf{S}^*\mathbf{S}^T + \mathbf{I}_{N_T}\right)\right] \\ &= N_R \log\left[\det\left(\sigma_n^{-2}\Sigma_G\left(\mathbf{U_H^H D U_H}\right)^* + \mathbf{I}_{N_T}\right)\right] \end{aligned}$$

B. Target Impulse Response Unknown to Transmitter

If the transmitter of MIMO radar A has no statistic of $\mathbf{G}$, the power is equally allocated to the transmitting antennas. The transmit waveform follows orthogonal design to satisfy $\mathbf{SS}^H = \mathbf{S}^*\mathbf{S}^T = P\mathbf{I}_{N_T}$. Orthogonal waveform design of MIMO radar can refer to [13]; on the other hand, orthogonal space-time coding can be used by the MIMO radar communication system [14]. In the case of equal power allocation, the radar mutual information is written as

$$I(\mathbf{G}; \Upsilon|\mathbf{S}) = N_R E\left\{\log\left[\det\left(\sigma_n^{-2}P\mathbf{G}\mathbf{G}^H + \mathbf{I}_{N_T}\right)\right]\right\} \quad (35)$$

At the same time, the communication mutual information is given by

$$C = N_S E\left\{\log\left[\det\left(\sigma_n^{-2}P\mathbf{H}^H\mathbf{H} + \mathbf{I}_{N_T}\right)\right]\right\} \quad (36)$$

C. Capacity With Maximum Radar MI

This section consider the radar-communication system, which firstly guarantees the radar performance and then takes account of the communication channel capacity. Assume that the statistic of $\mathbf{G}$ is known to the MIMO radar A, and eigenvector $\mathbf{U}_G$ is determined. Therefore, the water pouring power allocation strategy is found and reflected in the diagonal matrix $\mathbf{Q}$. As illustrated in Fig. 2, the communication symbol matrix is generated before radar waveform generater. Define a $N_T \times N_S$ matrix $\mathbf{X}$ which includes the communication symbols on N_T antennas during a pulse. The radar waveform is generated by precoding the communication symbol matrix as

$$\mathbf{S} = \mathbf{U}_G^*\mathbf{Q}^{1/2}\mathbf{X} \quad (37)$$

According to (25), if $\mathbf{XX}^H = \mathbf{\Psi}^H\mathbf{\Psi} = \mathbf{I}_{N_T}$, the water pouring strategy remains the same and the radar mutual information endures no loss. Let the (i, j)th element of $\mathbf{X}$ being $x_{i,j}$, which is information-bearing and drawn from a constellation $C = \{c_l,\ l = 1, \cdots, L\}$. As we know, orthogonal space-time codewords for real constellations can be designed for MIMO communication systems with any number of transmit antennas [14]. On the condition of orthogonal design of $\mathbf{X}$, we have

$$\begin{aligned} E\left\{\mathbf{SS}^H\right\} &= \mathbf{U}_G^*\mathbf{Q}^{1/2}\mathbf{X}\left(\mathbf{U}_G^*\mathbf{Q}^{1/2}\mathbf{X}^H\right)^H \quad (38) \\ &= \mathbf{U}_G^*\mathbf{Q}^{1/2}\mathbf{\Psi}^T\mathbf{\Psi}^*\mathbf{Q}^{1/2}\mathbf{U}_G^H \\ &= \mathbf{U}_G^*\mathbf{Q}\mathbf{U}_G^H \end{aligned}$$

Assume that $h_{i,j}$'s, the elements of channel transfer matrix $\mathbf{H}$, are independent zero-mean circularly symmetric complex Gaussian variables. The channel input is $\mathbf{S}$, which contains the optimal power allocation strategy for radar. The ergodic capacity of MIMO radar self-communication system is calculated as

$$\begin{aligned} C &= E\left\{N_S \log\left[\det\left(\sigma_n^{-2}\Sigma_S\mathbf{H}^H\mathbf{H} + \mathbf{I}_{N_T}\right)\right]\right\} \quad (39) \\ &= E\left\{N_S \log\left[\det\left(\sigma_n^{-2}\mathbf{U}_G^*\mathbf{Q}\mathbf{U}_G^H\mathbf{H}^H\mathbf{H} + \mathbf{I}_{N_T}\right)\right]\right\} \end{aligned}$$

VI. Simulations

In this section, the mutual information of MIMO radar and MIMO radar self-communication system is evaluated by simulations. The target echo impulse response matrix and the channel transfer matrix are generated according to the complex Gaussian pdf using MATLAB. In simulations, $N_S = N_R N_T$. All the results below are averaged over 2000 runs.

Firstly, the influence of antenna number on the radar mutual information and the channel capacity of self communication system is studied. In the simulation, the number of the transmit antenna is equal to the number of received antenna, i.e. $N_R = N_T$. In Figure 3, radar mutual information is obtained through optimal power allocation, while the capacity of radar self-communication system is achieved on the condition of given optimal radar mutual information, i.e. (39). Radar mutual information with SNR being 0dB is lower than the radar mutual information with SNR being 10dB, so is the capacity of radar self-communication. Note that radar mutual information is always lower than the capacity of radar self-communication

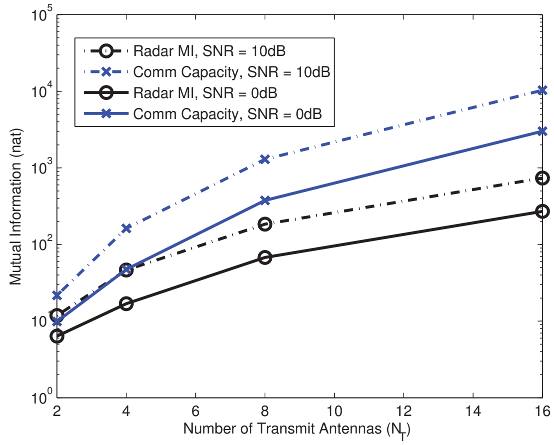

Fig. 3. The mutual information of MIMO radar and the capacity of MIMO radar self-communication channel versus the number of antennas.

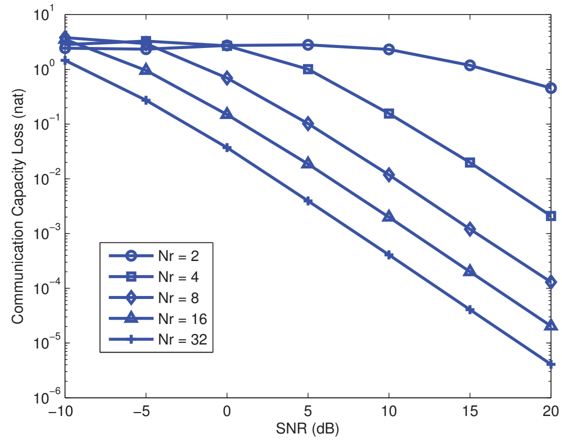

Fig. 5. Capacity loss of communication channel under the condition of optimal radar mutual information.

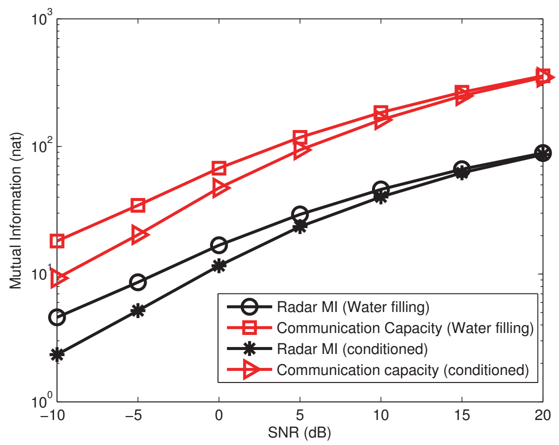

Fig. 4. The mutual information of MIMO radar and the capacity of MIMO radar self-communication channel versus SNR.

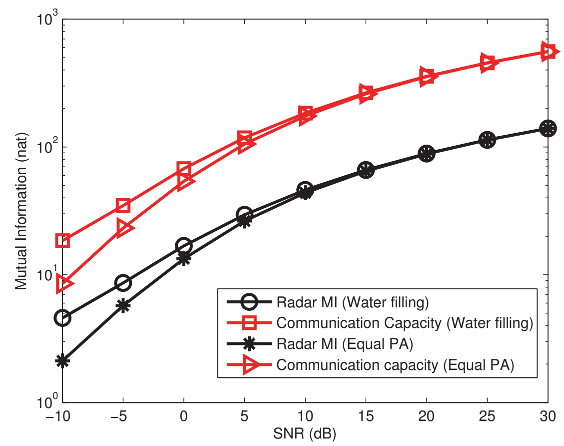

Fig. 6. Comparison between the water pouring power allocation and equal power allocation.

due to the difference of the baseband signal model leading to the different mutual information. Radar mutual information and capacity increase with the number of antennas.

Secondly, the interaction between the two functions is investigated. One pair of statistics to be compared is the optimal radar mutual information and the channel capacity of radar self-communication on the condition that radar achieves the optimal mutual information. The other pair of statistics to be compared is the optimal capacity of radar self-communication and the conditional radar mutual information on the condition that radar self-communication achieves the optimal power allocation. For the latter comparison, the optimal capacity of radar self-communication is calculated using (32), while the conditional radar mutual information is given by (35). Fig. 4 shows two pairs of statistics. Only 4 antennas are used at both ends of the transmitter and the receiver. It is obvious that the optimal radar mutual information is larger than the conditioned radar mutual information, and the capacity of radar self-communication is higher than the conditioned radar self-communication capacity. The loss between the optimal radar mutual information and the conditioned radar mutual information is observed. But in the high SNR region, the mutual information loss is not obvious.

We also show the capacity loss of the radar self-communication. In this case, the number of transmit antennas is 2, while the number of receive antennas varies from 2 to 32. Fig. 5 gives the loss between the capacity of communication-first system and the conditional capacity of radar self-communication. It is shown that the loss decreases with the increasing of SNR. Especially, in the high SNR region, the loss becomes more and more smaller.

Lastly, the optimal water pouring strategy and the equal power allocation strategy are compared. In practice, the transmitter has no knowledge about the target echo impulse response or the channel state information, and the transmitting power is equally allocated onto the antennas. Fig. 6 presents the comparison of capacity with water pouring power allocation and equal power allocation. The radar mutual information with water pouring and equal power allocation are given too. The number of antennas is 4. In the low SNR region, water pouring strategy is better than equal power allocation strategy. Note that $\mathbf{G}$ and $\mathbf{H}$ are generated by circularly symmetric complex Gaussian pdf. If $\mathbf{G}$ and $\mathbf{H}$ are with the other statistic characteristic, the gap between the two strategies will be different.

VII. Conclusion

MIMO radar self-communication system shares the hardware and frequency spectrum with the MIMO radar, and modulates the radar baseband waveform with the communication symbols. To guarantee the radar performance unaffected, the channel capacity of radar self-communication system is constrained by the optimal power allocation, which leads to maximum radar mutual information. As same as the pure MIMO communication system, the channel capacity of MIMO radar self-communication increases with increasing SNR and also with the number of antennas on either side of the channel. Compared with the optimal power allocation for MIMO communication without considering the radar performance, the MIMO radar self-communication system endures the capacity loss due to the constraint of optimal radar power allocation. In the high SNR region, the capacity loss is not obvious; while in the low SNR region, the capacity loss is modest. Therefore, we are convinced that the MIMO radar self-communication can meet the demand of radar networking.

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