Distributed Load Scheduling in Smart Community With Capacity Constrained Local Power Supplier

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***Abstract*—In this paper, we investigate the residential load scheduling problem within a smart energy community, which is powered by a primary utility along with a small scale local power supplier. As a premise, unit prices set by these two suppliers are different and both are time-varying. Therefore, users are motivated to control their household appliances' operation time and calculate appropriate portions of power purchased from these two suppliers to achieve bill curtailments. The capacity constraint of local power supplier, arising from the renewable energy source and the limited storage capability, also should not be violated. We formulate a residential load scheduling problem to address this situation. Distributed scheme based on information exchange among users is proposed, without over revealing individual user's load profile. Then we propose a distributed algorithm to solve this scheduling problem. Simulation results show that the proposed approach can reduce energy cost of the community and cut down electricity payments of users, and the peak-to-average ratio in load demand is also decreased.**

I. Introduction

Demand response management (DRM) is one of the key components of smart grid [1]. Electric utility companies (UCs) adopt DRM to efficiently manage power supplies and encourage users to change their electricity usage patterns to reduce peak load. It helps to cut down the cost of power generation and reduce the electricity bills of users, and also provides promising approaches to enhance efficiency and reliability of the whole power system [2].

Residential load scheduling is important in DRM to manage home users' power consumptions and to stabilize power load profiles. Load scheduling with single utility company (UC) has been intensively studied in literature [3]–[8]. The presented methods could be divided into two categories: centralized and distributed scheduling. Centralized load scheduling was used to achieve a trade-off between the minimization of electricity payments and the minimization of waiting time for operation of each appliance [3]. Users' comfort settings and load uncertainties were also considered in [4], [5]. Distributed load scheduling approach was proposed to optimize electricity consumptions across multiple users [6]. To address interactions among users, distributed algorithms based on game theory analyses were proposed to achieve peak load reduction as well as bill curtailments for users [7], [8].

As renewable energy sources increase, more than one power supplier usually coexist in the same region. Therefore, it is necessary to study the DRM problem with multiple UCs. A few recent works discussed multi-utility DRM problems [9]–[11]. In [9], a Stackelberg game was proposed between the UCs and the users for optimal price setting and optimal power consumption respectively. To reduce both the peak load and the variations of power demand, the authors of [10] and [11] applied game analysis and dual decomposition approaches to address the competition among UCs and each user's response to the differentiated prices. However, none of them considered the scheduling problem at the level of power consumption of each household appliance.

In this paper, we focus on the residential load scheduling problem with multiple UCs and multiple users. We consider a smart energy community made up of residential users, which is powered by a primary utility (PU). Meanwhile, users can also get access to the power produced by a small scale local power supplier (LPS), such as a photovoltaic system constructed by a third-party company [12]. Assume that the unit prices set by the PU and the LPS are different and both are time-varying. Users should decide when to operate their household appliances and how much power to purchase from these two suppliers, to save their electric bills. Moreover, the LPS relies on the renewable energy source and has limited power storage ability. That results in the capacity constraint of LPS, which should be taken into account when the power consumption scheduling strategy is implemented.

The contributions of this paper are summarized as follows:

1) We formulate a residential load scheduling problem in a scenario that users within a community are powered by two suppliers (i.e., PU and LPS).
2) We propose a distributed scheme based on information exchange among users, without over revealing individual user's load profile.
3) We develop a distributed algorithm to minimize the total energy cost of the community, as well as to reduce the electricity payment of each user, in the condition of limited power generation capability of LPS.

The remainder of this paper is organized as follows. System model is described in Section II. In section III, we present the residential load scheduling problem formulation and analysis. Distributed algorithm for solving the problem is proposed in Section IV. Simulation results are given in Section V and the paper is concluded in Section VI.

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II. System Model

A. Smart Energy Community

Consider a smart energy community composed by a number of residential users and a neighborhood coordinator (NC). The community is powered by a PU along with a LPS through the power line. We assume that each user has a smart meter embedded with an energy management controller (EMC) [7]. The role of EMC is to communicate with the NC for coordination of power consumptions in the community, and to control the operation time of each appliance. Although the introduction of NC makes the scheme not fully distributed, it's imperative for privacy preservations and reasonable dispatching of electricity from LPS. The details of interactions between the NC and users will be presented in section IV. Communications among all participants (i.e., users, NC and two suppliers) rely on an infrastructure such as a neighborhood area network (NAN) that introduced in [13]. A schematic diagram of this system is depicted in Fig. 1.

Notice that, in general, the buying price set by the PU is much lower than its selling price. It's reasonable for the LPS to charge users at an appropriate price between that two prices. Then, all users are motivated to trade with the LPS to make benefits. In addition, due to high cost of investment and inefficiency of power storage, the LPS can afford only a limited storage capacity. In excess of electricity produced by the LPS in certain time interval, the remaining power would be sold to the PU. On the contrary, when the LPS is short of supply, users need to buy electricity from the PU.

B. Energy Cost Model

We consider the cycle of a day as one period, which is divided into a set $\mathcal{H} \triangleq \{1, \dots, H\}$ of time slots. Without loss of generality, we set one slot as an hour, i.e., $H = 24$. Let $\mathcal{N} \triangleq \{1, \dots, N\}$ denote a set of users. And for each user $n \in \mathcal{N}$, let g_n^h and r_n^h denote the load allocated to the PU and to the LPS at hour $h \in \mathcal{H}$, respectively. The daily load of user n is divided into two portions denoted by vectors:

$$\boldsymbol{g}_n \triangleq \left[g_n^1, \dots, g_n^H\right], \quad \boldsymbol{r}_n \triangleq \left[r_n^1, \dots, r_n^H\right]. \tag{1}$$

The aggregate load across all users at each hour h of the day could be calculated as:

$$g^h = \sum_{n \in \mathcal{N}} g_n^h, \quad r^h = \sum_{n \in \mathcal{N}} r_n^h, \quad \forall h \in \mathcal{H}, \tag{2}$$

for PU and LPS.

From the perspective of power suppliers, there are many practical energy cost functions to reflect the cost of power generation to serve users. We assume that the PU uses *quadratic cost function* [14]. Then the cost of PU at each hour h is given by:

$$f_g^h(g^h) = a_h (g^h)^2 + b_h g^h + c_h, \quad \forall h \in \mathcal{H}, \tag{3}$$

where $a_h > 0$ and $b_h, c_h \geq 0$. For the LPS, we consider two kinds of cost functions. One is the *quadratic cost function*. In this case, the cost of LPS at each hour h is:

$$f_r^h(r^h) = a'_h (r^h)^2 + b'_h r^h + c'_h, \quad \forall h \in \mathcal{H}, \tag{4}$$

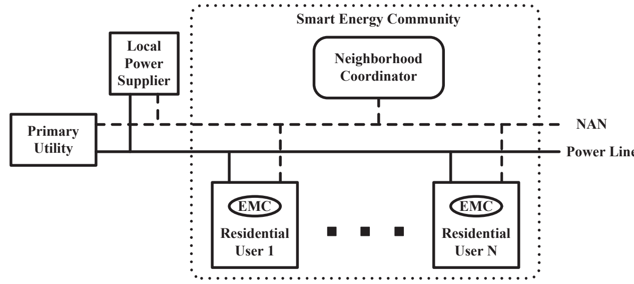

Fig. 1. Block diagram of a smart energy community with two power suppliers.

where $a'_h > 0$ and $b'_h, c'_h \geq 0$. The other is the *linear cost function*. In this case, the cost of LPS at each hour h is:

$$f_r^h(r^h) = a''_h r^h + b''_h, \quad \forall h \in \mathcal{H}, \tag{5}$$

where $a''_h > 0$ and $b''_h \geq 0$. Therefore, the energy cost of all users at each hour h is:

$$C^h(\cdot) = f_g^h(g^h) + f_r^h(r^h), \quad \forall h \in \mathcal{H}. \tag{6}$$

Note that the previously mentioned cost functions (3)-(4) are increasing and strictly convex, and cost function (5) is increasing and convex [15]. Such that, cost function (6) is increasing and strictly convex.

C. Residential Load Scheduling

For each user n, let $\mathcal{A}_n$ denote the set of household appliances with shiftable load ability, e.g., water heaters, dishwashers, wash machines and dress dryers. For each appliance $a \in \mathcal{A}_n$, there are two power sources (i.e., PU and LPS). We use two power consumption scheduling vectors:

$$\boldsymbol{x}_{n,a,g} \triangleq \left[x_{n,a,g}^1, \dots, x_{n,a,g}^H\right], \tag{7}$$

and

$$\boldsymbol{x}_{n,a,r} \triangleq \left[x_{n,a,r}^1, \dots, x_{n,a,r}^H\right] \tag{8}$$

to denote the daily power that scheduled to be purchased from PU and LPS, where scalars:

$$x_{n,a,g}^h \geq 0, \quad x_{n,a,r}^h \geq 0, \quad \forall h \in \mathcal{H} \tag{9}$$

are power consumption schedules at each hour h. Then, the power from PU to each user n at each hour h is:

$$g_n^h = \sum_{a \in \mathcal{A}_n} x_{n,a,g}^h, \quad \forall n \in \mathcal{N}, \forall h \in \mathcal{H}, \tag{10}$$

and the power from LPS to each user n at each hour h is:

$$r_n^h = \sum_{a \in \mathcal{A}_n} x_{n,a,r}^h, \quad \forall n \in \mathcal{N}, \forall h \in \mathcal{H}. \tag{11}$$

We set a pre-defined daily power demand $E_{n,a}$ for an appliance $a \in \mathcal{A}_n$. And a time interval $\mathcal{H}_{n,a} = [\alpha_{n,a}, \beta_{n,a}]$ is selected, where $\alpha_{n,a}$ and $\beta_{n,a}$ are the beginning and the end of it, such that appliance a could be scheduled. That imposes certain constraints on scheduling vectors $\boldsymbol{x}_{n,a,g}$ and $\boldsymbol{x}_{n,a,r}$:

$$\sum_{h=\alpha_{n,a}}^{\beta_{n,a}} \left(x_{n,a,g}^h + x_{n,a,r}^h\right) = E_{n,a} \tag{12}$$

and

$$x^h_{n,a,g} = 0, \quad x^h_{n,a,r} = 0, \qquad \forall h \in \mathcal{H} \setminus \mathcal{H}_{n,a}. \tag{13}$$

A household appliance $a \in \mathcal{A}_n$ has certain minimum standby power level $\underline{x}_{n,a}$ and maximum power level $\overline{x}_{n,a}$, i.e.,

$$\underline{x}_{n,a} \leq x^h_{n,a,g} + x^h_{n,a,r} \leq \overline{x}_{n,a}, \qquad \forall h \in \mathcal{H}. \tag{14}$$

The electricity produced by LPS at each hour is denoted by R^h for all $h \in \mathcal{H}$. Then, the load demand dispatched to the LPS from all users should not exceed the supply of LPS at each hour. That imposes following constraint:

$$r^h = \sum_{n\in\mathcal{N}} r^h_n = \sum_{n\in\mathcal{N}} \sum_{a\in\mathcal{A}_n} x^h_{n,a,r} \leq R^h, \qquad \forall h \in \mathcal{H}. \tag{15}$$

Generally, the electricity produced by LPS with renewable energy source at each hour is variable. However, we assume that $R^h, \forall h \in \mathcal{H}$ is already known through power generation forecasting methods [16].

The power consumption schedule for each user $n \in \mathcal{N}$ in a day is denoted by a vector:

$$\boldsymbol{x}_n \triangleq \{\boldsymbol{x}_{n,a,g}, \boldsymbol{x}_{n,a,r}, \forall a \in \mathcal{A}_n\}, \tag{16}$$

which indicates the operation time and power consumptions of all its appliances. A schedule is feasible only when constraints (9), (12)-(15) are satisfied.

Technical informations of four typical household appliances and schedulable time intervals for the operations of these appliances are listed as input data for simulation in Section V.

III. Problem Formulation and Analysis

A. Tariff Strategy

The tariff strategy of a power supplier should cover the cost of power generation to make profits, and be fair among users to make it practicable. We assume that both the PU and the LPS adopt the billing system proposed in [8].

For each user $n \in \mathcal{N}$, let Ω_n denote the proportion of power consumed by user n, which is calculated by:

$$\Omega_n \triangleq \frac{\sum_{a\in\mathcal{A}_n} E_{n,a}}{\sum_{n=1}^{N} \sum_{a\in\mathcal{A}_n} E_{n,a}}. \tag{17}$$

Then, the daily payment of electricity for user n is:

$$p_n = \kappa \Omega_n C_{total}(\cdot), \tag{18}$$

where $\kappa \geq 1$ indicate the profit ratio of the power supplier. That means users are charged proportional to their daily power consumptions, and also on the total energy cost of the community. Then users are encouraged to change their load profiles to reduce the energy cost of the community as well as to save their electric bills.

B. Centralized Cost Minimization

Given κ and $\Omega_n, \forall n \in \mathcal{N}$, our goal is to achieve minimum energy cost of the whole community by scheduling the power consumption of each appliance. The total energy cost of users in a day is calculated as:

$$\begin{aligned} C_{total}(\cdot) &= \sum_{h=1}^{H} C^h(\cdot) = \sum_{h=1}^{H} \left(f^h_g(g^h) + f^h_r(r^h)\right) \\ &= \sum_{h=1}^{H} \Bigg(f^h_g\Big(\sum_{n\in\mathcal{N}} \sum_{a\in\mathcal{A}_n} x^h_{n,a,g}\Big) \\ &\quad + f^h_r\Big(\sum_{n\in\mathcal{N}} \sum_{a\in\mathcal{A}_n} x^h_{n,a,r}\Big)\Bigg). \end{aligned} \tag{19}$$

Therefore, the residential load scheduling problem can be formulated as an optimization problem:

$$\begin{aligned} \min_{\boldsymbol{x}_n, \forall n\in\mathcal{N}} \quad & C_{total}(\cdot) \\ \text{s.t.} \quad & (9), (12)\text{-}(15) \end{aligned} \tag{20}$$

Explanations of constraints in problem (20) are followed. The first constraint (9) implies that the power consumption of each appliance in each hour should not be negative. Constraints (12)-(14) are operation requirements of all appliances, and they also indicate load allocations to power suppliers. Constraint (15) is due to the limited power generation capacity of LPS.

The optimization problem (20) is convex, for all cost functions (3)-(6) are increasing and convex. It could be solved in a centralized manner by the NC using convex optimization techniques [15] or some other methods [17]. Additionally, since (6) is strictly convex, problem (20) always has an unique solution.

C. Decentralized Cost Minimization

Although the NC can play a role of central controller to solve the centralized optimization problem (20) within a small-scale community, this approach is not scalable as the number of users increases. In addition, it requires all users to report operation parameters of their appliances to the NC. In this set-up, working status of each load in each hour is on NC's decision. Moreover, detailed running informations of all appliances are disclosed. It will be an obstacle for users to get involved in the demand response (DR) programs, when privacy issues are concerned. Therefore, solving this optimization problem in a decentralized way is desired.

Check (19) again, we can rewrite it as

$$\begin{aligned} C'_{total}(\cdot) = \sum_{h=1}^{H} \Bigg(& f^h_g\Big(\sum_{a\in\mathcal{A}_n} x^h_{n,a,g} + \sum_{m\in\mathcal{N}\setminus\{n\}} g^h_m\Big) \\ & + f^h_r\Big(\sum_{a\in\mathcal{A}_n} x^h_{n,a,r} + \sum_{m\in\mathcal{N}\setminus\{n\}} r^h_m\Big)\Bigg). \end{aligned} \tag{21}$$

Then we can formulate a local optimization problem for each user $n \in \mathcal{N}$:

$$\begin{array}{ll} \min\limits_{\boldsymbol{x}_n} & C'_{total}(\cdot) \\ \text{s.t.} & (9), (12)\text{-}(15) \end{array} \tag{22}$$

Although the objective function of problem (22) is exactly the same as the one of problem (20), only a few local variables are needed to be decided for each user herein. The size of local problem remains a constant, while the scale of community increases. Given aggregate daily load profiles of all other users, i.e., $\sum_{m\in\mathcal{N}\setminus\{n\}} g_m^h$ and $\sum_{m\in\mathcal{N}\setminus\{n\}} r_m^h$, it's easy for each user $n \in \mathcal{N}$ to handle the local problem solution.

From (21), we note that each user's decision of load schedule depends on power consumptions of all other users. That leads to form the following *power consumption game* among users:

- Players: Residential users in set $\mathcal{N}$.
- Strategies: Each user $n \in \mathcal{N}$ decides its power consumption schedule vector $\boldsymbol{x}_n$ to maximize its payoff.
- Payoffs: $P_n(\boldsymbol{x}_n; \boldsymbol{x}_{-n})$ for each user $n \in \mathcal{N}$, where

$$\begin{aligned} P_n(\boldsymbol{x}_n; \boldsymbol{x}_{-n}) &= -p_n \\ &= -\kappa\Omega_n C'_{total}(\cdot). \end{aligned} \tag{23}$$

In (23), $\boldsymbol{x}_{-n} \triangleq [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{n-1}, \boldsymbol{x}_{n+1}, \ldots, \boldsymbol{x}_N]$ denotes the vector of power consumption schedules of all users other than user n.

In this game, each user in the community act as a player, with a strategy to select the daily power consumption vector $\boldsymbol{x}_n$, to maximize the payoff function $P_n(\boldsymbol{x}_n; \boldsymbol{x}_{-n})$.

Theorem 1. *The Nash equilibrium of the power consumption game is exists and unique.*

Proof. Since $C^h(\cdot)$ is strictly convex in each hour $h \in \mathcal{H}$, then $C'_{total}(\cdot)$ is also strictly convex. Thus, given the $\boldsymbol{x}_{-n}$, the payoff function $P_n(\boldsymbol{x}_n; \boldsymbol{x}_{-n})$ is strictly concave with respect to $\boldsymbol{x}_n$. Therefore, the power consumption game is a strictly concave *n-person game*. In this condition, an unique Nash equilibrium is exists according to the theorems given in [18]. □

Theorem 2. *The unique Nash equilibrium of the power consumption game is the optimal solution of the energy cost minimization problem* (20)*.*

Proof. Let $\{\boldsymbol{x}_1^*, \ldots, \boldsymbol{x}_N^*\}$ denote the optimal solution of problem (20), then the optimal total energy cost is calculated by:

$$\begin{aligned} C^*_{total}(\cdot) = \sum_{h=1}^{H} \Bigg(& f_g^h\Big(\sum_{n\in\mathcal{N}}\sum_{a\in\mathcal{A}_n} x_{n,a,g}^{h*}\Big) \\ & + f_r^h\Big(\sum_{n\in\mathcal{N}}\sum_{a\in\mathcal{A}_n} x_{n,a,r}^{h*}\Big)\Bigg). \end{aligned} \tag{24}$$

For each user $n \in \mathcal{N}$, given any arbitrary $\boldsymbol{x}_n \geq 0$, we have:

$$\begin{aligned} C^*_{total}(\cdot) \leq \sum_{h=1}^{H} \Bigg(& f_g^h\Big(\sum_{a\in\mathcal{A}_n} x_{n,a,g}^{h} + \sum_{m\in\mathcal{N}\setminus\{n\}} g_m^{h*}\Big) \\ & + f_r^h\Big(\sum_{a\in\mathcal{A}_n} x_{n,a,r}^{h} + \sum_{m\in\mathcal{N}\setminus\{n\}} r_m^{h*}\Big)\Bigg), \end{aligned} \tag{25}$$

where $g_m^{h*} = \sum_{a\in\mathcal{A}_m} x_{m,a,g}^{h*}$ and $r_m^{h*} = \sum_{a\in\mathcal{A}_m} x_{m,a,r}^{h*}$, $\forall m \in \mathcal{N}\setminus\{n\}, \forall h \in \mathcal{H}$. By definition of payoff function of the game, we get:

$$P_n(\boldsymbol{x}_n^*; \boldsymbol{x}_{-n}^*) \geq P_n(\boldsymbol{x}_n; \boldsymbol{x}_{-n}^*), \quad \boldsymbol{x}_n \geq 0. \tag{26}$$

From (26), we can see that the optimal solution $\{\boldsymbol{x}_1^*, \ldots, \boldsymbol{x}_N^*\}$ forms a Nash equilibrium for the power consumption game. According to Theorem 1, the power consumption game has an unique Nash equilibrium. Thus, the optimal solution of problem (20) is equivalent to the Nash equilibrium of the power consumption game. □

From Theorem 1 and 2, when the power consumption game is at its Nash equilibrium, no user would benefit by deviating from the optimal power consumption schedule $\boldsymbol{x}_n^*, \forall n \in \mathcal{N}$. Therefore, the users can solve (22) locally and cooperate with each other to minimize the energy cost of the whole community, and thus to cut down their own electricity bills. Details of the corresponding algorithm will be illustrated in Section IV.

IV. Distributed Algorithm

In this section, we focus on the distributed algorithm executed by the NC and users. Interactions between these two parts are stated first, followed by descriptions of the algorithm. Then, convergence and optimality of the algorithm are discussed.

A. Interactions Between NC and Users

From the mathematical formulation of local optimization problem (22), we note that power consumption informations of all other users are required for an user to get a solution. It could be realized through broadcasting in the community. But the communication overhead is inevitable and the privacy issues occur. Furthermore, the capacity constraint of LPS (15) is coupled among users. When local optimization is being carried out, each user must make sure that there is no other user synchronized. Otherwise, constraint (15) is probably violated. Because of these two reasons, there should be an equipment to coordinate the information exchange and sequence the local optimization in the community. That's why the NC is introduced.

The NC is regarded as a neutral and trustful agency. It could be implemented as a dedicated device or be integrated with the data aggregation unit in future smart grid infrastructure. The NC gets daily load vector (1) from each user and sends back the sum of all other users' load vectors in turn. Such that, each user only needs to reveal the load vector, not the detailed operation information of each appliance, to the NC. And each user has no way to explore the power consumption patterns of other users from the aggregate load informations that received. Therefore, the coordination of users and the privacy concerns are taken into account.

B. Algorithm Descriptions

For notation simplicity to illustrate the algorithm, we denote the vector containing daily aggregate load dispatched to the PU from all users other than user n as:

$$\boldsymbol{g}_{-n} = \left[\sum_{m \in \mathcal{N} \setminus \{n\}} g_m^1, \ldots, \sum_{m \in \mathcal{N} \setminus \{n\}} g_m^H \right], \tag{27}$$

and the corresponding vector for the LPS as

$$\boldsymbol{r}_{-n} = \left[\sum_{m \in \mathcal{N} \setminus \{n\}} r_m^1, \ldots, \sum_{m \in \mathcal{N} \setminus \{n\}} r_m^H \right]. \tag{28}$$

Then, the algorithms executed by the NC and the EMC of each user are summarized in **Algorithm 1** and **2**.

Algorithm 1 is executed by the NC. As the optimization procedure starts, a list of users that involved in the DR program is constructed as the input, and each user's daily load vectors are initialized as all zeros. At each iteration from line 2 to line 9, an user in the list is selected, the NC sums up all other users' load vectors and sends the result to this user. Then, the user solves the local problem (22). Once a new power consumption schedule obtained, the user responds to the NC with renewed load vectors. Such that, all users update their load vectors sequentially. If any user does not respond with an update or a no-update message before the timer expired for some reasons, e.g., communication congestions in the network, it will be skipped in this iteration. The procedure will stop when no user changes the load vectors, or the maximum number of iterations reached.

Algorithm 1 : Executed by the NC.

Input: List of users.
Initialize: $\boldsymbol{g}_n \leftarrow \boldsymbol{0}$ and $\boldsymbol{r}_n \leftarrow \boldsymbol{0}$ for each user $n \in \mathcal{N}$.

```
1:  repeat
2:    for n = 1 to N do
3:      Set a timer.
4:      Send g_{-n} and r_{-n} to user n.
5:      Wait for user n to execute Algorithm 2.
6:      if Update message received from user n before the
        timer expired. then
7:        Update g_n and r_n accordingly.
8:      end if
9:    end for
10: until No user sends update message or the maximum
    number of iterations reached.
```

Algorithm 2 is implemented in the EMC of each user and invoked by the NC. Hourly power generation of the LPS, operation parameters of appliances and the aggregate load vectors of all other users are required inputs. As the procedure starts, local problem (22) is solved by convex optimization methods. If the power consumption schedule changed, an update message packaged with daily load vectors is fed back to the NC. Otherwise, a no-update message is sent to the NC.

Algorithm 2 : Executed by the EMC of each user $n \in \mathcal{N}$.

Input: Capacity constraint $R^h, \forall h \in \mathcal{H}$ of the LPS , operation parameters of appliances, and vectors of $\boldsymbol{g}_{-n}$ and $\boldsymbol{r}_{-n}$ received from the NC.
Initialize: $\boldsymbol{x}_n \leftarrow \boldsymbol{0}$.

```
Solve local optimization problem (22).
if Energy consumption schedule x_n changed. then
  Update x_n.
  Send update message containing g_n and r_n to the NC.
else
  Sent no-update message to the NC.
end if
```

C. Convergence and Optimality

Consider the algorithm proposed in Section IV-B. Although it is implemented in a distributed way, the whole process is dominated by the NC. At the first iteration, users report their load vectors to the NC one after another. As starts from the initial condition of zero, total energy cost of the community increased gradually with users' costs added up. After that, users aim to reduce the total cost by recalculating their power consumption schedules asynchronously. Once an user is selected to solve the local optimization problem, a new solution is obtained to decrease the value of total cost. Otherwise, the schedule is the same with the result of previous iteration and the cost remains unchanged. The descending value of total cost, combined with a lower bound of zero, insures that the Algorithm 1 and 2 converge to a fixed point.

At the fixed point of Algorithm 1 and 2, no user can decrease the total cost further by changing the schedule. In other words, no user can improve its payoff by deviating from the fixed point. That means the set of all users' decisions at this point is a Nash equilibrium of the *n-person game* among users [19]. According to Theorems 1 and 2, this Nash equilibrium is equivalent to the optimal solution of the energy cost minimization problem (20). Therefore, our proposed distributed algorithm converges to the global optimum after a number of iterations.

V. Simulation Results

In this section, we evaluate the performance of our proposed distributed algorithm within an example smart energy community. The simulation is implemented in MATLAB environment with the optimization problem solving package CVX [20].

A. Simulation Setup

In the considered smart energy community, there are $N = 20$ residential users. Each user has 8 to 10 appliances in two categories, i.e., shiftable and non-shiftable appliances.

TABLE I

TYPICAL OPERATION PARAMETERS OF SHIFTABLE APPLIANCES

Appliance	E_a	$[\alpha_a, \beta_a]$	$\underline{x}_a$	$\overline{x}_a$
Water heater	2.4	[0:00 AM,12:00 PM]	0.05	1.0
Dishwasher	1.44	[8:00 PM,6:00 AM]	0.02	0.9
Wash machine	1.2	[8:00 AM,8:00 PM]	0.02	0.8
Dress dryer	1.5	[7:00 PM,7:00 AM]	0.05	1.0

Shiftable appliances can be scheduled by the EMC in predetermined time intervals, while non-shiftable appliances have strict power consumption constraints. Four shiftable appliances (i.e., water heater, dishwasher, wash machine and dress dryer) are considered in the simulation.

Typical operation parameters of these appliances are listed in Table I, where E_a, $\underline{x}_a$ and $\overline{x}_a$ indicate daily power usage, minimum and maximum power level of appliance a in unit of kWh. The operational time interval for an appliance is denoted by $[\alpha_a, \beta_a]$. Non-shiftable appliances include lighting ($E_a =$ 0.9 kWh), refrigerators ($E_a = 1.2$ kWh), TVs ($E_a = 0.5$ kWh), microwave ovens ($E_a = 0.5$ kWh), air conditioners ($E_a = 4.5$ kWh), etc. Although power consumptions of non-shiftable appliances can not be changed at each hour, their loads that dispatched to different suppliers (i.e., PU and LPS) can be decided by the EMC. Total load of no-shiftable appliances, which is essential in daily life, is set to be greater than the total load of shiftable appliances in the simulation.

In addition, we set the parameters of cost functions. For the PU, we assume that $b_h = c_h = 0$ for all $h \in \mathcal{H}$, $a_h = 0.2$ cents from 8:00 AM to 12:00 PM and $a_h = 0.15$ cents from 12:00 PM to 8:00 AM the day after. For the LPS, there are two cases: 1) when the quadratic cost function is adopted, we set $b'_h = c'_h = 0$ for all $h \in \mathcal{H}$, $a'_h = 0.15$ cents from 8:00 AM to 12:00 PM and $a'_h = 0.1$ cents from 12:00 PM to 8:00 AM the day after; 2) when the linear cost function is adopted, we set $b''_h = 0$ for all $h \in \mathcal{H}$, $a''_h = 7.5$ cents from 8:00 AM to 12:00 PM and $a''_h = 5$ cents from 12:00 PM to 8:00 AM the day after. Profit ratio of each supplier is set to $\kappa = 1$.

For the capacity of LPS, we suppose that 200 sets of KD220GX-LPU photovoltaic modules are installed [21]. Therefore, the power generation of LPS at each hour from 10:00 AM to 6:00 PM is assumed to be 20 kWh, 42 kWh, 60 kWh, 80 kWh, 80 kWh, 60 kWh, 42 kWh and 20 kWh, respectively.

B. Performance Evaluation of Distributed Algorithm

To evaluate the performance of our proposed distributed algorithm, three different cases are simulated and compared. For the first case, no scheduling scheme is implemented. Loads from users are allocated to the LPS until the capacity constraint reached, then the remaining part of loads is covered by the PU. For the other two cases, centralized and distributed scheduling algorithms are carried out.

To investigate the impacts of two different cost functions that considered for the LPS, we run the simulations when the LPS adopts the quadratic cost function and the linear cost function, respectively.

TABLE II

SIMULATION RESULTS WITH QUADRATIC COST FUNCTION OF THE LPS

Scheduling scheme	Total cost	Avg. payment	PAR
no	145.69$	7.28$	2.18
centralized	120.96$	6.05$	2.01
distributed	120.96$	6.05$	2.01

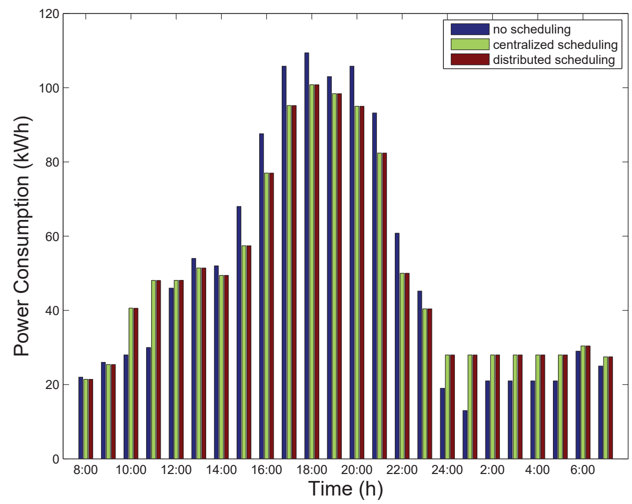

Fig. 2. Power consumptions at each hour for three cases with quadratic cost function of the LPS.

1) LPS Adopts the Quadratic Cost Function

In Table II, we summarize the total energy cost of community, the average daily bill payment of users and the peak-to-average ratio (PAR) of power consumption in a day for the three cases. As we can see, when scheduling strategies are enabled, the total energy cost is reduced from 145.69$ to 120.96$ (i.e., 16.97% less) and the average bill payment is reduced from 7.28$ to 6.05$ (i.e., 16.90% less). The PAR is reduced from 2.18 to 2.01 with 7.80% less. It is noteworthy that the distributed scheme has exactly the same performance as the centralized one does, which verifies the convergence and optimality of our proposed distributed algorithm.

In Fig. 2, power consumption at each hour of a day for three cases are depicted. It shows that, when centralized and distributed load scheduling schemes are applied, peak-hour loads are reduced and shifted to the night when the electricity price is lower, and also to the hours when the power supply of LPS is available. That leads to the smoother load profiles.

Fig. 3 gives the energy cost at each hour of a day in different cases. We can see that, for each case, most of the energy cost is incurred when the load is high and the power from LPS is not available. We also note that, in each hour from 12:00 AM to 11:00 PM, the energy cost is greatly reduced when centralized/distributed algorithm is applied. That results in significant energy cost saving in a day.

To show the utilization of the power from LPS, we compare

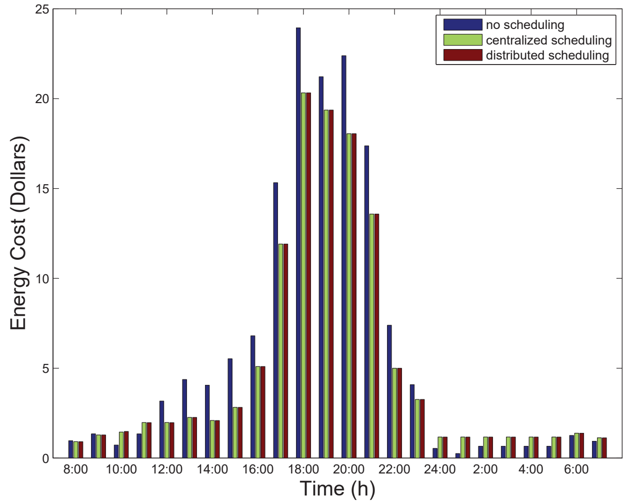

Fig. 3. Energy cost at each hour for three cases with quadratic cost function of the LPS.

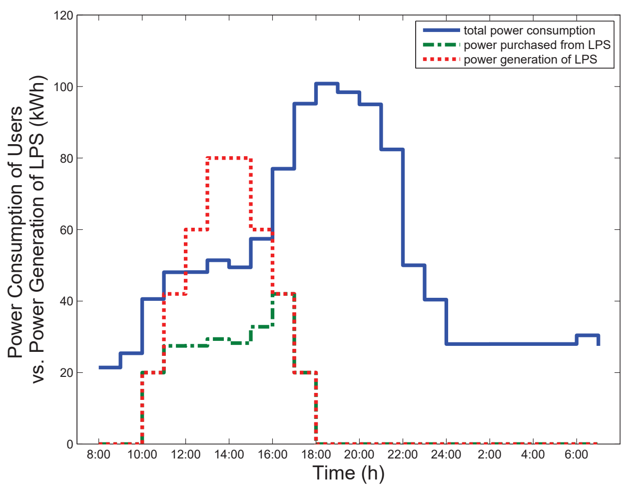

Fig. 4. Power consumption of users and power purchased from LPS in comparison with power generation of LPS, where quadratic cost function is adopted by the LPS.

the aggregate power consumption of users and the power purchased from LPS with the power generation of LPS in Fig. 4. As can be seen, the peak hours (i.e., from 1:00 PM to 4:00 PM) of power generation of the LPS are not just correspond to the peak load hours (i.e., from 6:00 PM to 9:00 PM). The extra power of LPS is sold to the PU at a much lower price. Enlargement of power storage capability will not only help the LPS to supply more power at a higher price but also contribute to the peak load alleviation for the PU. While the trade-off between profit increments and additional investment costs should be considered. Moreover, even when the capacity of LPS exceeds the aggregate load of users, there is still a part of loads dispatched to the PU. That is due to the tariff strategy adopted by the LPS, which charges the users based on a quadric function of power consumption. Although the unit price of LPS is lower, the value of cost function

TABLE III
SIMULATION RESULTS WITH LINEAR COST FUNCTION OF THE LPS

Scheduling scheme	Total cost	Avg. payment	PAR
no	147.79$	7.39$	2.18
centralized	127.66$	6.38$	2.00
distributed	127.66$	6.38$	2.00

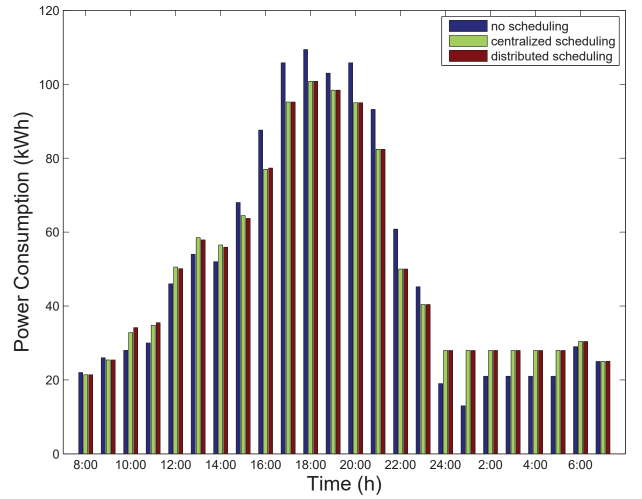

Fig. 5. Power consumptions at each hour for three cases with linear cost function of the LPS.

increases drastically as more loads are allocated to the LPS. Therefore, the scheduling program will seek to the supply of PU to minimize the total cost.

2) LPS Adopts the Linear Cost Function

Table III summarizes the simulation results when the linear cost function is adopted by the LPS. It shows that, when scheduling strategies are enabled, the total energy cost is reduced from 147.79$ to 127.66$ (i.e., 13.6% less) and the average bill payment is reduced from 7.39$ to 6.38$. The PAR is reduced from 2.18 to 2.00 with 8.26% less.

Fig. 5 gives the power consumptions at each hour of a day for three different cases. It shows that, the load profiles are smoothed by the centralized and distributed load scheduling schemes. Fig. 6 depicts the energy costs at each hour of a day. We can see that, the energy costs in peak hours are greatly reduced by the scheduling algorithm. That demonstrates our proposed algorithm can achieve significant energy cost saving when the LPS adopts different kinds of cost functions.

Fig. 7 shows the aggregate power consumption of users versus the power generation of the LPS at each hour of a day. When the capacity of LPS exceeds the aggregate load of users, there is still a part of loads dispatched to the PU. Compared with Fig. 4, where the quadratic cost function is adopted by the LPS, the utilization of power that generated by the LPS is different. That implies the type of cost function and the parameter settings of cost function have significant impacts on the power consumptions of users. To fully utilize the power generation of the LPS, alternative cost function of

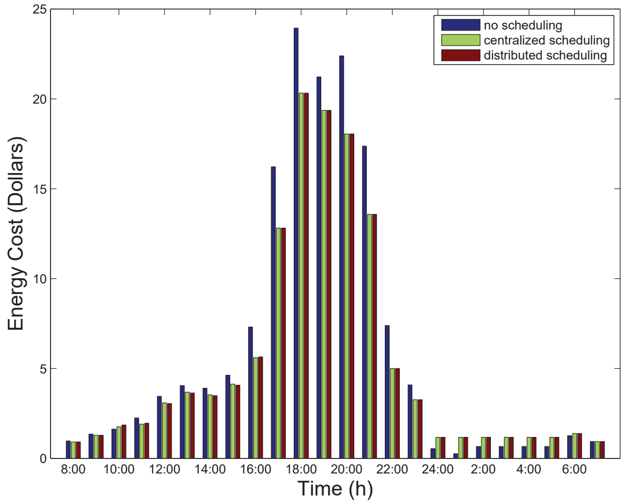

Fig. 6. Energy cost at each hour for three cases with linear cost function of the LPS.

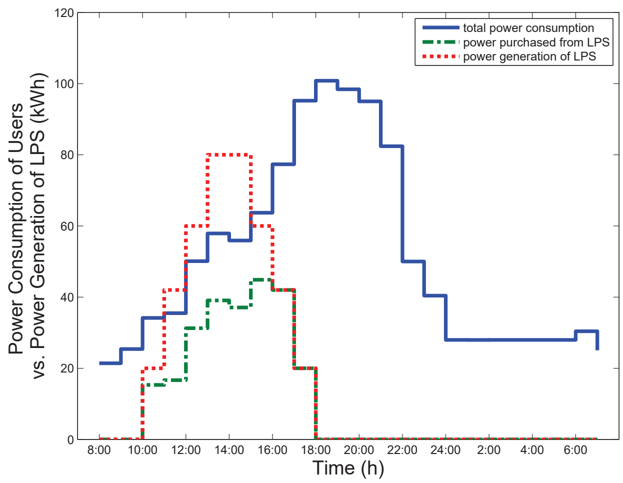

Fig. 7. Power consumption of users and power purchased from LPS in comparison with power generation of LPS, where linear cost function is adopted by the LPS.

the LPS should be considered, and the pricing strategy of the system also should be revised. That will be investigated in our future works.

VI. Conclusion

In this paper, we considered the residential load scheduling problem in a smart energy community that powered by two suppliers. We proposed a distributed algorithm to solve this problem. With the coordination scheme implemented, each user obtained the optimal power consumption schedule to minimize its bill payment as well as the total cost of the community. Simulation results showed that our proposed scheduling scheme achieved significant energy cost saving and payment reduction compared with the situation when no scheduling scheme was applied, and it also resulted in the reduction of peak-to-average ratio in load demand.

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