Optimum Reference Node Deployment for Indoor Localization Based on the Average Mean Square Error Minimization

Fei Long, Aydin Behnad, and Xianbin Wang
Department of Electrical and Computer Engineering
Western University, London, Ontario, Canada
{flong4, abehnad, xianbin.wang}@uwo.ca

***Abstract*—Using the Global Positioning System (GPS) for indoor localization is challenging as the GPS signal is significantly attenuated or completely blocked by the building. In achieving indoor localization, a set of transmitters from wireless communication networks are often used as reference nodes to localize target nodes with unknown locations. The accuracy of such localization process is considerably affected by the spatial distribution of the reference nodes with respect to the target node to be localized. Hence, deploying the reference nodes at best locations will notably increase the localization precision. In this paper, a novel reference nodes deployment scheme is introduced which minimizes the average Mean Square Error (MSE) of the localization over the area of interest with certain shape. The proposed scheme is evaluated for deploying the reference nodes in the circular, square, and hexagonal localization regions. The proposed scheme is validated and illustrated by numerical simulations.**

I. Introduction

Location information of a node in a wireless network is valuable for enhancing the communications performance as well as in achieving location-based services. Nevertheless, there are many challenging problems for wireless network based localization in a large scale [1]. An immediate solution for locationing is to use the GPS. However, this approach fails in many scenarios because of the GPS signal blockage, e.g., indoor environment, or is not cost-effective when nodes are manufactured in a very large scale, e.g., wireless sensor networks. In indoor environment, there are generally three signal parameters based on which location of a target node and internodal distances are determined: received signal strength (RSS), time of arrival (ToA) or time difference of arrival (TDoA), and angle of arrival (AoA) [2]. The RSS model works based on the fact that the transmitted signal is attenuated as a function of the distance between the target node and the reference transmitter. In ToA and TDoA, the signal propagation time between the pair of nodes (ToA) or the time difference of receiving the same transmitted signal by different nodes (TDoA) is measured and translated into distances. The AoA is based on the angular direction under which a signal is received and the distances are obtained using geometrical relations between the target node and reference nodes.

Considering all the localization systems, a critical common issue needs to be tackled is to determine the physical positions of reference nodes. In [3], it shows that the placement of reference nodes has substantial impact on the positioning accuracy. Therefore, finding the optimum locations for reference nodes are vital in nearly every localization systems and reference nodes placement strongly affects the quality of localization. Many localization methods proposed have been proposed recently to find the optimum reference nodes to reduce the localization estimation error. In [4], reference anchors are optimized to improve the performance of linear least squares and weighted least squares algorithm in RSS-based localization.

In wireless networks, Cramer-Rao Bound (CRB) is usually defined as a benchmark so that we can know how good an estimator will possibly do [1]. It also helps us compare the performances between different localization estimations [5]. Average CRB is derived in [6] based on RSS localization to study the nonlinear least square solution presented. Another TOA-based average CRB in none-line-of-sight environment can be found in [7]. According to [7] and [8], CRB provides a lower bound on the MSE of a position estimate. In some cases, average MSE is used to evaluate practical estimators' performance in positioning systems. A reference node selection strategy is proposed in [9] which minimizes the average mean square error (MSE) in the TDoA based position system. In this paper, we are interested in the first two diagonal terms of the CRB which provide the minimum mean square error (MMSE) of the position estimation. We propose a reference nodes deployment scheme which determines the optimum locations by minimizing MMSE over selected areas which can be circle, hexagon or square. Besides, our scheme can also be used as the guidance for deploying reference nodes in localization system.

The organization of this paper is as follows. In Section II, the preliminaries and background theories are introduced. Section III presents the system model and derives the expressions of MMSE criteria in different scenarios. Section IV gives out the simulation results and section V comes to the conclusion.

II. Preliminaries and Theoretical Background

Among all techniques used for localization, RSS, AoA, and ToA/TDoA, are the most common schemes and have extensively used for different localization scenarios [10]. RSS is the simplest method of distance measurement based on

978-1-4673-8590-9/15/$31.00 ©2015 IEEE

the relation between the transmitted signal attenuation with distance; however, its accuracy is not good due to the multipath fading effect of the environment. Also the positioning error is a multiplicative factor of the range and increases proportional to the distance [6]. The AoA scheme is more complicated than the RSS and requires either a directional antenna, or an array of antennas for each node to find the direction of arrival. This scheme is also affected by the multipath fading effect. The ToA/TDoA techniques are more complex than the RSS but their measurement errors are not multiplicative as RSS and it does not require any additional resources such as directional antenna similar to AoA. Here, we also consider the ToA based localization network in this paper.

As mentioned above, ToA is based on the signal propagation time measurement between nodes. If $(x_{\mathrm{T}}, y_{\mathrm{T}})$ is the location of a target node T being localized and (x_i, y_i) is the location of the i-th reference node localizing T,

$$d_i(x,y) = \sqrt{(x_{\mathrm{T}} - x_i)^2 + (y_{\mathrm{T}} - y_i)^2} \tag{1}$$

is the distance between T and the i-th reference node. In this case, the signal propagation time is measured as

$$t_i = \frac{d_i}{c} + \zeta_i, \quad i = 1, ..., N, \tag{2}$$

where constant c is the signal propagation speed, and ζ_i is the measurement noise which is usually modeled as a zero-mean Gaussian random variable with a constant variance σ_i^2 [1], and N is the number of reference nodes. In this case, the location of T is obtained based on the measured time values $(t_1, t_2, \ldots, t_N)$. In addition, if $(\hat{x}_{\mathrm{T}}, \hat{y}_{\mathrm{T}})$ is the estimated location of T, then the localization error is obtained as

$$e = \sqrt{(\hat{x}_{\mathrm{T}} - x_{\mathrm{T}})^2 + (\hat{y}_{\mathrm{T}} - y_{\mathrm{T}})^2}, \tag{3}$$

which can be used to compare the accuracy between different localization estimators.

In addition to the accurate estimation of the distances between the target node and any of the reference nodes, the topology of the reference nodes with respect to the target node can also significantly affect the performance of the target node localization. Hence, in order to get the best estimation of the target node, it is more efficient to use the reference nodes with optimum locations for locating the target node.

The average mean square error of e derived in (3) is a good figure of merit to select the best set of the reference nodes for the localization. Using a CRB approach, it has been shown in [7] and [8] that the minimum mean square error (MMSE) of the ToA localization scheme is given by

$$MMSE_{TOA} = \frac{c^2 \Sigma_{i=1}^{N_L} \sigma_i^{-2}}{\Sigma_{i=1}^{N_L} \Sigma_{j=1}^{i-1} \sigma_i^{-2} \sigma_j^{-2} \sin^2(\theta_i - \theta_j)}, \tag{4}$$

where c is the speed of the light, σ_i and σ_j represent the ith and jth TOA measurement errors respectively, N_L is the number of reference nodes. $\theta_i = \tan^{-1}(\frac{y - y_i}{x - x_i})$ and it denotes the angle between the ith reference node and the target node. (x, y) and (x_i, y_i) are the positions of the target node and the ith reference node, respectively.

In order to get the average MMSE within a certain range, we have to get the average MMSE expression. The TOA-based average MMSE result can be expressed as

$$MMSE(x,y) = \frac{c^2 \sigma^2 N}{\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} (\sin\theta_{ij})^2}, \tag{5}$$

where $\theta_{ij} = \theta_i - \theta_j$ and N is the number of reference nodes. σ^2 denotes the TOA measurement error.

In the usual situation where has 3 reference nodes , we can simplify the average MMSE expression from (5) to

$$MMSE(r,\theta) = \frac{3c^2\sigma^2}{\sin^2\theta_{12} + \sin^2\theta_{23} + \sin^2\theta_{13}}, \tag{6}$$

where θ_{12}, θ_{23} and θ_{13} denote the angular distances between every two reference nodes as shown in Fig.1. The target node is in the center of the figure presented as triangle and the 3 reference nodes are randomly distributed which are circles in Fig. 1. In this paper, we set $c\sigma$ equal to 1. Also, $\sin^2\theta_{12}$, $\sin^2\theta_{23}$, $\sin^2\theta_{13}$ can be derived by the vector product expression

$$\mathbf{a} \times \mathbf{b} = \| \mathbf{a} \| \| \mathbf{b} \| \sin\theta, \tag{7}$$

where $\mathbf{a}$ and $\mathbf{b}$ are the vectors of segments connect the target node and the reference node.

$$\sin^2\theta_{12} = \frac{(x_{\mathrm{A}}y_{\mathrm{C}} - x_{\mathrm{C}}y_{\mathrm{A}})^2}{(x_{\mathrm{C}}^2 + y_{\mathrm{C}}^2)(x_{\mathrm{A}}^2 + x_{\mathrm{C}}^2 - 2x_{\mathrm{A}}x_{\mathrm{C}} + y_{\mathrm{C}}^2 + y_{\mathrm{A}}^2 - 2y_{\mathrm{A}}y_{\mathrm{C}})}, \tag{16}$$

$$\sin^2\theta_{23} = \frac{(x_{\mathrm{B}}y_{\mathrm{C}} + x_{\mathrm{A}}y_{\mathrm{B}} + x_{\mathrm{C}}y_{\mathrm{A}} - x_{\mathrm{B}}y_{\mathrm{A}} - x_{\mathrm{A}}y_{\mathrm{C}} - x_{\mathrm{C}}y_{\mathrm{B}})^2}{(x_{\mathrm{B}}^2 + x_{\mathrm{C}}^2 - 2x_{\mathrm{B}}x_{\mathrm{C}} + y_{\mathrm{C}}^2 + y_{\mathrm{B}}^2 - 2y_{\mathrm{B}}y_{\mathrm{C}})(x_{\mathrm{A}}^2 + x_{\mathrm{C}}^2 - 2x_{\mathrm{A}}x_{\mathrm{C}} + y_{\mathrm{C}}^2 + y_{\mathrm{A}}^2 - 2y_{\mathrm{A}}y_{\mathrm{C}})}, \tag{17}$$

$$\sin^2\theta_{13} = \frac{(x_{\mathrm{B}}y_{\mathrm{C}} - x_{\mathrm{C}}y_{\mathrm{B}})^2}{(x_{\mathrm{C}}^2 + y_{\mathrm{C}}^2)(x_{\mathrm{B}}^2 + x_{\mathrm{C}}^2 - 2x_{\mathrm{B}}x_{\mathrm{C}} + y_{\mathrm{B}}^2 + y_{\mathrm{C}}^2 - 2y_{\mathrm{B}}y_{\mathrm{C}})}. \tag{18}$$

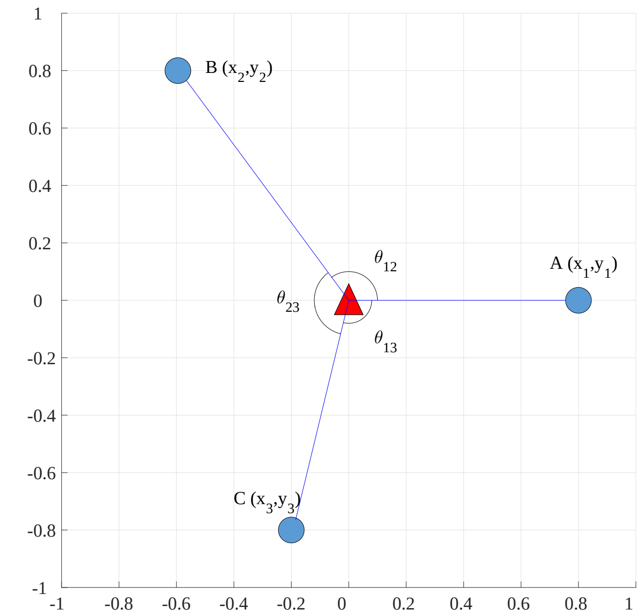

Fig. 1. Illustration of 3 Reference Nodes in the Mean Square Error Minimization Problem

III. System Model

We consider a three-reference-node localization scenario to localize a target node T located in a determined localization region. We analyze three different cases separately. In the first case, one reference node is assumed to be fixed at the center of the localization region, which with refer to it as the base station, whereas the other two nodes are deployed to help the base station to localize T which minimize the localization error over this region.

At first, we assume the nodes are in an unit circle. In this circular region, there are one reference node O with known location in the center and two reference nodes A, B with unknown location within the circle, then the parameters in average MMSE expression from (6) can be calculated out as

$$\sin^2\theta_{12} = \frac{r_\text{A}^2 \sin^2\theta}{r_\text{A}^2 + r^2 - 2rr_\text{A}\cos\theta}, \tag{8}$$

$$\sin^2\theta_{23} = \frac{(rr_\text{B}\sin(\theta-\theta_0) - rr_\text{A}\sin\theta + r_\text{A}r_\text{B}\sin\theta_0)^2}{(r^2 + r_\text{B}^2 - 2rr_\text{B}\cos(\theta-\theta_0))(r_\text{A}^2 + r^2 - 2rr_\text{A}\cos\theta)} \tag{9}$$

and

$$\sin^2\theta_{13} = \frac{r_\text{B}^2\sin^2(\theta-\theta_0)}{r^2 + r_\text{B}^2 - 2rr_\text{B}\cos(\theta-\theta_0)}. \tag{10}$$

Here, r_A and r_B are the distances between O and A, B respectively. r is the distance between T and O. θ is the angular distance between the segments which connect A, T with O. θ_0 denotes the angular distance between the segments which connect A, B with O.

The average MMSE result within this unit circle is considered as

$$\overline{MMSE} = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 MMSE(r,\theta|r_\text{A}, r_\text{B}, \theta_0) r dr d\theta. \tag{11}$$

In this way, the optimum locations of reference nodes A, B can be obtained by minimizing the average MMSE expression.

$$(\hat{r}_\text{A}, \hat{r}_\text{B}, \hat{\theta}_0) = \text{argmin}\,\overline{MMSE} \tag{12}$$

In this case, we also consider the square region as well as the hexagonal region to prove the flexibility of our scheme. For these two regions, the parameters of average MMSE expression in (6) are (16), (17) and (18).

The average MMSE expression within a square whose side is 1 is as follow:

$$\overline{MMSE} = \int_{-\frac{1}{2}}^{\frac{1}{2}}\int_{-\frac{1}{2}}^{\frac{1}{2}} MMSE(x_\text{C}, y_\text{C}|x_\text{A}, y_\text{A}, x_\text{B}, y_\text{B}) dxdy. \tag{19}$$

Similarly, the average MMSE expression within a hexagon whose side is 1 is as follow:

$$\overline{MMSE} = \frac{2\sqrt{3}}{9}\Big(\int_{-1}^{1}\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} MMSE(x_\text{C}, y_\text{C}|x_\text{A}, y_\text{A}, x_\text{B}, y_\text{B}) dxdy - 4\int_{\frac{1}{2}}^{1}\int_{-\sqrt{3}y+\sqrt{3}}^{\frac{\sqrt{3}}{2}} MMSE(x_\text{C}, y_\text{C}|x_\text{A}, y_\text{A}, x_\text{B}, y_\text{B}) dxdy\Big). \tag{20}$$

By minimizing the average MMSE result, we can get the positions of optimum reference nodes by

$$(\hat{x}_\text{A}, \hat{y}_\text{A}, \hat{x}_\text{B}, \hat{y}_\text{B}) = \text{argmin}\,\overline{MMSE} \tag{21}$$

in square region or in hexagonal region.

In the second case, we consider a situation where there are two reference nodes A and B with known location at central axis of a region. In circular region, we set A to (1,0) or ($\frac{1}{3}$,0) and B to (-1,0) or (-$\frac{1}{3}$,0) respectively. C is the reference node with unknown location within the region that we want to obtain. In this assumption, the parameters in (6) are

$$\sin^2\theta_{12} = \frac{(rr_\text{B}\sin\theta - rr_\text{A}\sin\theta)^2}{(r_\text{A}^2 + r^2 - 2rr_\text{A}\cos\theta)(r_\text{B}^2 + r^2 - 2rr_\text{B}\cos\theta)}, \tag{22}$$

$$\sin^2\theta_{23} = \frac{(r_\text{A}r_\text{C}\sin\theta_0 - rr_\text{A}\sin\theta + rr_\text{C}\sin(\theta-\theta_0))^2}{(r^2 + r_\text{C}^2 - 2rr_\text{C}\cos(\theta-\theta_0))(r_\text{A}^2 + r^2 - 2rr_\text{A}\cos\theta)}, \tag{23}$$

and

$$\sin^2\theta_{13} = \frac{(r_\text{B}r_\text{C}\sin\theta_0 - rr_\text{B}\sin\theta + rr_\text{C}\sin(\theta-\theta_0))^2}{(r_\text{B}^2 + r^2 - 2rr_\text{B}\cos\theta)(r^2 + r_\text{C}^2 - 2rr_\text{C}\cos(\theta-\theta_0))}. \tag{24}$$

Similarly, in square and hexagonal region, the parameters in (6) are

$$\sin^2\theta_{12} = \frac{(x_\text{B}y_\text{C} - x_\text{B}y - xy_\text{C} + x_\text{C}y)^2}{[(x_\text{B}-x)^2 + y^2][(x_\text{C}-x)^2 + (y_\text{C}-y)^2]}, \tag{25}$$

$$\sin^2\theta_{23} = \frac{(x_\text{A}y_\text{C} - x_\text{A}y - xy_\text{C} + x_\text{C}y)^2}{[(x_\text{A}-x)^2 + y^2][(x_\text{C}-x)^2 + (y_\text{C}-y)^2]} \tag{26}$$

and

$$\sin^2\theta_{13} = \frac{(x_\text{A}y - x_\text{B}y)^2}{(x_\text{B}^2 + x^2 - 2xx_\text{B} + y^2)(x_\text{A}^2 + x^2 - 2xx_\text{A} + y^2)}. \tag{27}$$

$$\sin^2\theta_{12} = \frac{((r_{\rm B}\cos\theta_{\rm B} - r_{\rm T}\cos\theta_{\rm T})(r_{\rm C}\sin\theta_{\rm C} - r_{\rm T}\sin\theta_{\rm T}) - (r_{\rm B}\sin\theta_{\rm B} - r_{\rm T}\sin\theta_{\rm T})(r_{\rm C}\cos\theta_{\rm C} - r_{\rm T}\cos\theta_{\rm T}))^2}{(r_{\rm B}^2 + r_{\rm T}^2 - 2r_{\rm B}r_{\rm T}\cos(\theta_{\rm B} - \theta_{\rm T}))(r_{\rm C}^2 + r_{\rm T}^2 - 2r_{\rm C}r_{\rm T}\cos(\theta_{\rm C} - \theta_{\rm T}))} \quad (28)$$

$$\sin^2\theta_{23} = \frac{((r_{\rm B}\cos\theta_{\rm B} - r_{\rm T}\cos\theta_{\rm T})(-r_{\rm T}\sin\theta_{\rm T}) - (r_{\rm B}\sin\theta_{\rm B} - r_{\rm T}\sin\theta_{\rm T})(r_{\rm A} - r_{\rm T}\cos\theta_{\rm T}))^2}{(r_{\rm B}^2 + r_{\rm T}^2 - 2r_{\rm B}r_{\rm T}\cos(\theta_{\rm B} - \theta_{\rm T}))(r_{\rm A}^2 + r_{\rm T}^2 - 2r_{\rm A}r_{\rm T}\cos(\theta_{\rm A} - \theta_{\rm T}))} \quad (29)$$

$$\sin^2\theta_{13} = \frac{((r_{\rm A}\cos\theta_{\rm A} - r_{\rm T}\cos\theta_{\rm T})(r_{\rm C}\sin\theta_{\rm C} - r_{\rm T}\sin\theta_{\rm T}) - (r_{\rm A}\sin\theta_{\rm A} - r_{\rm T}\sin\theta_{\rm T})(r_{\rm C}\cos\theta_{\rm C} - r_{\rm T}\cos\theta_{\rm T}))^2}{(r_{\rm A}^2 + r_{\rm T}^2 - 2r_{\rm A}r_{\rm T}\cos(\theta_{\rm A} - \theta_{\rm T}))(r_{\rm C}^2 + r_{\rm T}^2 - 2r_{\rm C}r_{\rm T}\cos(\theta_{\rm C} - \theta_{\rm T}))} \quad (30)$$

But in hexagonal region, A and B are set to be $(\frac{\sqrt{3}}{6}, 0)$ and $(-\frac{\sqrt{3}}{6}, 0)$ or $(\frac{\sqrt{3}}{2}, 0)$ and $(-\frac{\sqrt{3}}{2}, 0)$.

Actually in reality, there is no reference node with known location in advance for many cases. So at last, we consider the third case where we want to obtain three reference nodes A, B and C without prior location information. We present the parameters in (6) as (28), (29) and (30) in circular region where $\theta_{\rm A}$, $\theta_{\rm B}$, $\theta_{\rm C}$, $\theta_{\rm T}$ denote the angle between the x-axis through the center of the circle and segment which connects A, B, C, T with the center of the circle, respectively. In square and hexagonal region, the parameters in (6) are

$$\sin^2\theta_{12} = \frac{[(x_{\rm B} - x_{\rm T})(y_{\rm C} - y_{\rm T}) - (y_{\rm B} - y_{\rm T})(x_{\rm C} - x_{\rm T})]^2}{[(x_{\rm B} - x_T)^2 + (y_{\rm B} - y_T)^2][(x_{\rm C} - x_{\rm T})^2 + (y_{\rm C} - y_{\rm T})^2]}, \quad (31)$$

$$\sin^2\theta_{23} = \frac{[(x_{\rm B} - x_{\rm T})(y_{\rm A} - y_{\rm T}) - (y_{\rm B} - y_{\rm T})(x_{\rm A} - x_{\rm T})]^2}{[(x_{\rm B} - x_{\rm T})^2 + (y_{\rm B} - y_{\rm T})^2][(x_{\rm A} - x_{\rm T})^2 + (y_{\rm A} - y_{\rm T})^2]} \quad (32)$$

and

$$\sin^2\theta_{13} = \frac{[(x_{\rm A} - x_{\rm T})(y_{\rm C} - y_{\rm T}) - (y_{\rm A} - y_{\rm T})(x_{\rm C} - x_{\rm T})]^2}{[(x_{\rm A} - x_{\rm T})^2 + (y_{\rm A} - y_{\rm T})^2][(x_{\rm C} - x_{\rm T})^2 + (y_{\rm C} - y_{\rm T})^2]}. \quad (33)$$

Because we have three reference nodes with unknown location in this assumed scenario, the optimum position information can be obtained by minimizing

$$(\hat{x}_{\rm A}, \hat{y}_{\rm A}, \hat{x}_{\rm B}, \hat{y}_{\rm B}, \hat{x}_{\rm C}, \hat{y}_{\rm C}) = \operatorname{argmin} \overline{MMSE}. \quad (34)$$

All the implements above will be simulated at one time and we will get the results at the same time.

Until now, we provide three common designs in three different shapes of region. This not only proves that our scheme is flexible but also gives a guidance for starting to set a localization system and obtain the optimum reference nodes placement.

IV. Simulation Results

In this section, we present the numerical optimization results for the three reference nodes positioning in different scenarios, based on the deployment region of the target node, as follows.

A. Circular Region

In Fig.2, the optimum deployment topologies are shown when the location of the reference nodes are unknown (Fig.2-(a)), one reference node is fixed at the center of the region (Fig.2-(b)), two reference nodes are fixed in the region (Fig.2-(c) and -(d)).

As shown in Fig.2-(a), assuming three reference nodes can be deployed anywhere in the circular region, the best structure is when the reference nodes have equiangular distances, i.e., equal to 120°, around the center of the region. Also, their distances to the center are equal, which is 0.901 times the radius of the region. Fig.2-(b) shows the optimum case when a reference node, i.e., one base station, is located at the center of the circular region and two other reference nodes, e.g., two access points or mobile users, cooperate with this base station for localization. As the figure depicts, the optimum positions for the two reference nodes are on the circumference

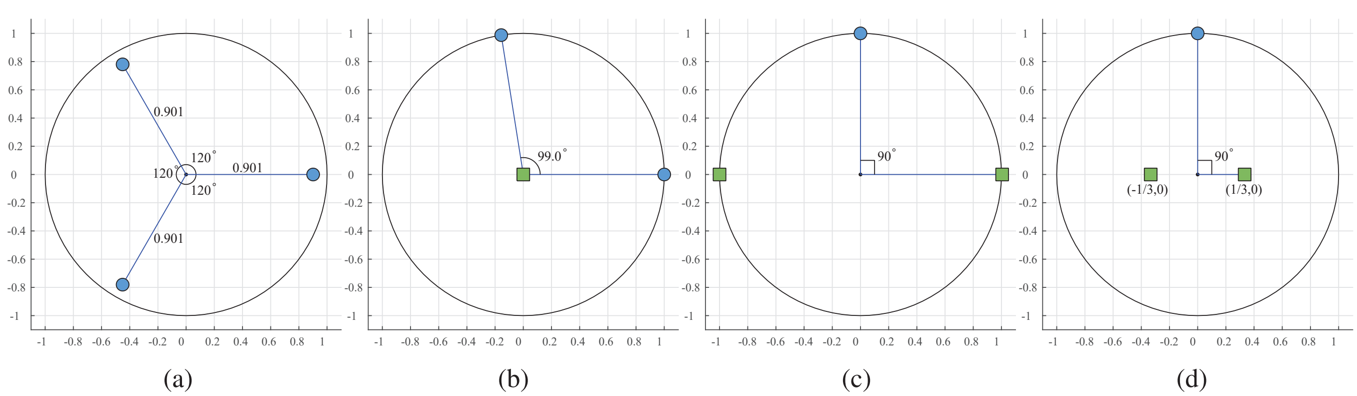

Fig. 2. Optimum RNs Selection Scheme Within Circular Region

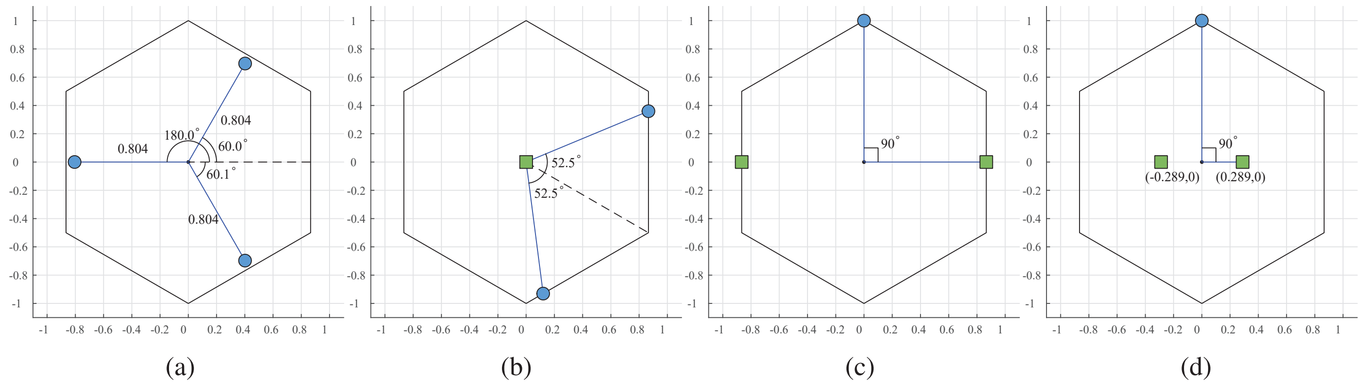

Fig. 3. Optimum RNs Selection Scheme Within Hexagonal Region

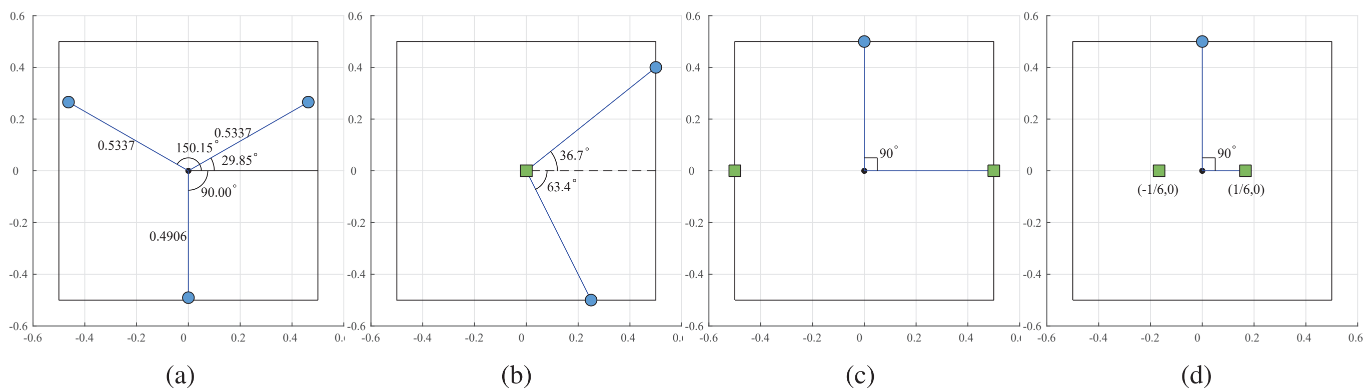

Fig. 4. Optimum RNs Selection Scheme Within Square Region

of the circle with angular distance $99.0°$ with respect to the center of the region. Fig.2-(c) and -(d) show the cases where two base stations are located on the diameter of the region symmetrically on two sides of the center, within and on the circumference of the circle, respectively. As seen, in both cases, the best position of the third reference node cooperating with the two base stations is on the intersection of the circumference of the circle and the perpendicular bisector of the base stations crossing line.

B. Hexagonal Region

Similar to the circular region case, the reference nodes placements shown in Fig.3 present three optimum localization conditions when no reference node is fixed (Fig.3-(a)), one reference node is fixed (Fig.3-(b)), and two reference nodes are fixed (Fig.3-(c) and -(d)).

For the first case where no reference node is fixed, the optimum locations of the three reference nodes are shown in Fig.3-(a). In this case, the reference nodes form an equilateral triangle within the hexagon which means they have equal angular distances $120°$ around the center of the region. In addition, the distances between the reference nodes and the center point are 0.804 times the side of the hexagonal region. In Fig.3-(b), a reference node as a base station is fixed at the center of hexagonal region and the other two reference nodes assist the base station to locate the target node in this region. From the result, we can find that the best positioning for the two reference nodes are at the boundary of the region with angular distance $105.0°$ with respect to the center point. In Fig.3-(c) and -(d), two base stations are locate on the central axis of the hexagonal region, symmetrically on two sides of the center point and separated one third of the diameter. The figures show that they have the same optimum reference node location which is at the middle of the region side which has equal distance to the base stations.

C. Square Region

In Fig.4, the optimum deployment topologies are shown for the square region, when the location of the three reference nodes are unknown (Fig.4-(a)), one reference node is fixed at the center of the region (Fig.4-(b)), and two reference nodes are fixed in the region (Fig.4-(c) and -(d)).

As depicted by Fig.4-(a), the optimum case when the the reference nodes can be located anywhere on the square region is similar to circular and hexagonal region cases, i.e., the angular separation between any two reference nodes is close to $120°$ and they maximally spread on the region. In addition, for the case where one reference node is fixed in the center of the region as a base station, the optimum location of the other two reference nodes are on the boundary of the square region with angular separation around $100°$, which is close to what have been obtained for the circular and hexagonal regions. Furthermore, for the two base station case where two reference nodes are fixed in the region or on the boundary, as shown by Fig.4-(a) and Fig.4-(b), respectively, the optimum location of the third reference node is similar to those of the circular and hexagonal regions, i.e., on the boundary and on the perpendicular bisector of the base stations crossing line.

In fact, our proposed scheme can be used in any shape of region and the number of the base stations or the reference nodes with no prior localization can also be different. Besides, when our scheme is applied to the situation where there are 3 unknown base stations, it can in fact be used to establish a localization system in any wireless network.

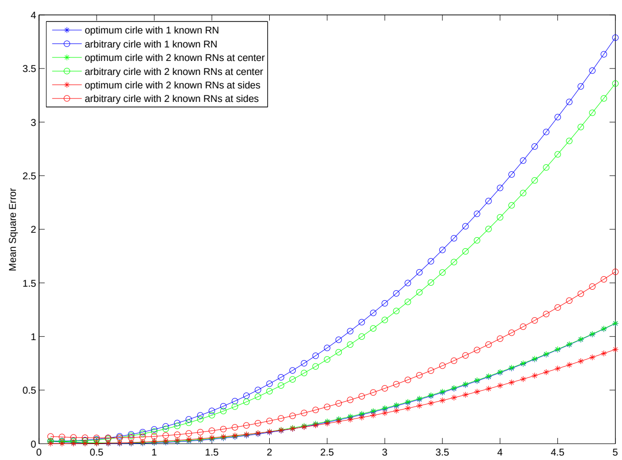

Fig. 5. Mean Square Error Comparison Between Optimum RNs and Arbitrary RNs in Circular Region

Fig.5 shows the 50-iteration simulation results which compare the situations when we use the selected optimum reference nodes according to our proposed scheme with the reference nodes randomly selected where we change location of one reference node among the optimum locations. In the simulation, we select four nodes which are in the center from four different quadrants as the target nodes and the distance between them and the central point is 0.5 in circular scenario. With iterations increasing, the variance of noise is increasing as well. As we can see in Fig.5, the accuracy can be increased by using our proposed reference nodes selection scheme.

V. Conclusion

In this paper, we propose a general scheme in determining the optimum reference nodes deployment scheme in circular, hexagonal and square locationing regions. The proposed method can also be applied in arbitrary region in realistic localization system. Besides, this scheme can be practical when extending an existed localization system. As the average positioning performance was carried out, the best locationing performance of the reference node deployment could be determined as the lowest MSE average value within a certain area. It was shown from the simulation results that the proposed scheme improves the accuracy of location.

References

[1] N. Patwari *et al.*, “Locating the nodes: Cooperative localization in wireless sensor networks,” *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 5469, Jul. 2005.

[2] R. Marcelin-Jimenez, M. Ruiz-Sanchez, Lopez-Villasenor, V. Ramos-Ramos, C. Moreno-Escobar, and M. Ruiz-Sandoval, *A Survey on Localization in Wireless Sensor Networks.* Hershey, PA, USA: IGI Global, 2010.

[3] O. Baala, Y. Zheng, and A. Caminada, “The impact of AP placement in WLAN-based indoor positioning system,” Proceedings of the 8th International Conference on Networks, Cancun, Mexico, pp. 1217, 2009.

[4] N. Salman, M. Ghogho, and A. H. Kemp, “Optimized low complexity sensor node positioning in wireless sensor networks,” *IEEE Sensors J.*, vol. 14, no. 1, pp. 3946, Jan. 2014.

[5] J. Shen, A. F. Molisch, and J. Salmi, “Accurate passive location estimation using TOA measurements,” *IEEE Transactions on Wireless Communications*,vol. 11, no. 6, pp. 2182-2192, Jun. 2012.

[6] X. Li, “RSS-based location estimation with unknown pathloss model,” *IEEE Transactions on Wireless Communications*, vol. 5, no. 12, pp. 3626-3633, Dec. 2006.

[7] Y. Qi, H. Kobayashi, and H. Suda, “Analysis of wireless geolocation in a non-line-of-sight environment,” *IEEE Transaction on Wireless Communications*, vol. 5, no. 3, pp.672-681, Mar. 2006.

[8] S. Gezici, “A Survey on Wireless Position Estimation,” *Wireless Pers. Commun.*, vol. 44, no. 3, pp. 26382, Feb. 2008.

[9] S. Monica and G. Ferrari, “UWB-based localization in large indoor scenarios: optimized placement of anchor nodes.” *IEEE Transactions on Aerospace and Electronic Systems*, vol. 51, no. 2, pp. 987-999, Apr. 2015.

[10] D. Fox, J. Hightower, and D. Schulz, “Bayesian Filtering for Location Estimation, *IEEE Pervasive Computing*, vol. 2, no. 3, pp. 24-33, July-Sept. 2002.