Pricing and Revenue Sharing in Secondary Market of Mobile Internet Access

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Abstract — There is a fast growing number of public spaces offering Wi-Fi access to meet the rising demands for Internet access. It is common for such service to be offered to users at no charge or for a flat fee. Both situations provide very little incentive for Wi-Fi providers to offer better service to the users. Similarly, Wi-Fi providers pay a monthly flat rate to ISP for Internet access and, this too does not incentivize ISP to offer better service to Wi-Fi users. As a result, Wi-Fi users may experience poor connection when network becomes congested during peak hours. In this paper we propose a dynamic pricing scheme for Internet access and a revenue sharing mechanism that provides incentives for both ISP and Wi-Fi providers to offer better service to their users. We build our revenue sharing model based on Shapley value mechanism. Importantly, our proposed revenue sharing mechanism captures the power negotiation between ISP and Wi-Fi providers, and how shifts in power influences revenue division. Specifically, the model assures that the party who contributes more receives a higher portion of the revenue. In addition, our simulation demonstrates that our model captures the bargaining power shifts between Wi-Fi providers and ISP, and shows that the division of revenue asymptotically converges to a percentage value.

I. INTRODUCTION

In the recent decade, wireless local area network technology (e.g. Wi-Fi) has become essential to our everyday activities including communication, commerce, entertainment, education, etc. To cater to the rising demand for Internet access, there is a growing number of public spaces offering Internet access, such as malls, airports, coffee shops, libraries, parks, etc. They are referred to as Wi-Fi providers (WFP). As this paper is written, WFPs commonly offer Internet access service free-of-charge or for a flat price. For example, a hotel charges their guest $5 per day for Internet access. Both free-of-charge or flat fee arrangements offer little economic incentives for WFPs to offer additional bandwidth to their users, when they need more bandwidth. This is because WFPs do not gain additional reward for providing better service to their users and this may result in users being stuck with the default service offered by WFP.

WFPs rely on Internet Service Providers (ISP) for the Internet access and pay ISPs a monthly flat rate for the service. This means ISPs are also responsible for the traffic generated by Wi-Fi users through WFPs. Similarly, due to the arrangement of monthly flat rate, there is also very little incentives for ISPs to provide additional bandwidth to Internet users of WFPs. Study [1] reports that Starbucks' customers experience poor connection whenever there is a high volume of customers in a coffee shop using the Internet at the same time. While the customers enjoy their drinks, they may be dissatisfied with their Internet service. To address these issues, we propose a pricing scheme and a revenue sharing mechanism that provide incentives to both ISPs and WFPs to offer better service to WFPs' users. In other words, we propose a mechanism that allows Wi-Fi users to pay more if they wish to attain more bandwidth. Otherwise, they can use the service that is available to them. Our contributions are the proposed pricing model and revenue sharing mechanism collectively address the rising concerns on network traffic load and price from ISPs, WFPs, and end users. Importantly, the revenue sharing mechanism assures economic incentives for both ISP and WFP while providing acceptable service to their users.

In this paper, we propose and develop a revenue sharing mechanism where the revenue from providing Internet access to users is shared between an ISP and a WFP. We build our revenue sharing model based on *Shapley value* mechanism [6,7] because of its capacity to divide the revenue "fairly" between parties involved. Our study shows that the cooperative scenario brought by our revenue sharing mechanism provides incentives to both an ISP and a WFP to offer additional bandwidth to the WFP's users. That is, our cooperative revenue sharing model ensures that an ISP receives a portion of the revenue collected by a WFP. This way, the ISP's revenue share increases as the WFP generates higher revenue. In addition, our revenue sharing model also discourages WFP's users from transmitting more data when an ISP is experiencing congestion at its end. Another important observation is that our sharing model also captures the shift in bargaining power between an ISP and a WFP according to the amount of revenue generated. Specifically, the revenue sharing model apportions a larger share of the revenue to an ISP when a WFP generates lower revenue, and this is reasonable because the ISP has to incur a minimum infrastructure overhead cost at all times. When a WFP generates higher revenue from its users, our model attributes a higher portion of the revenue in recognition of its more significant contribution. However, the division of revenue between a WFP and an ISP converges to a percentage value even when the WFP generates most of the revenue. The above outcomes confirm that our revenue sharing model

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achieves "fair" compensation to both a WFP and an ISP according to Shapley value characteristics. We also demonstrate that our revenue sharing also assures that both ISP and WFP have incentive to offer additional service even with dynamic number of users. At last, we also demonstrate a "best practice" for a WFP and an ISP to achieve higher shared revenue, while keeping users happy at the same time.

Next, we argue that dynamic pricing (other than flat rate and free service) is able to provide economic incentive to both ISP and WFP for offering additional bandwidth to their users. Following this argument, we design a dynamic pricing model that incentivizes both ISP and WFP to provide additional bandwidth. Our pricing model is built upon two considerations. The first consideration is that a dynamic pricing should adapt to the fluctuation of the level of bandwidth demand. Thus, an ISP needs to decide the minimum sale price by considering the total traffic load generated by the WFP as well as those generated by the ISP's other subscribers. Similarly, the WFP computes the price to its users according to the level of demand, i.e., price should increase as the level of demand increases. The second consideration is that the final price charged to a WFP's users must be equal or higher than the minimum price set by an ISP. This is because the final price should cover at least the ISP's minimum price. For instance, when the minimum price set by the ISP is higher than the price set by the WFP, then the WFP charges its users with price set by the ISP. Importantly, our study shows that this pricing model provides incentive for both an ISP and a WFP to offer additional bandwidth to their users. This pricing model follows the economic concept of demand and supply, where a WFP charges higher price for service when demand for access increases. We formulate our pricing scheme as a Network Utility Maximization (NUM) problem [2,3,4], and solve it using a subgradient based algorithm. The ISP's and the WFP's price are obtained from the solution to the NUM.

The rest of this paper is organized as follows. We begin by formulating our research problem in section II where we discuss the impact of pricing and revenue sharing on ISP, different types of pricing and revenue sharing approaches, and related works. In section III, we present our proposed revenue sharing mechanism between ISP and WFP. Following that, we present the dynamic pricing mechanism for both WFP and ISP in section IV. Our simulation results are presented in section V, followed by concluding remarks.

II. Background

A. Impact of WFP's traffic on ISP

In order to design suitable pricing and sharing mechanisms, the impact of traffic generated through WFPs on an ISP must be considered. Providing a Wi-Fi service in a public space may increase the amount of data being injected into the network. With the additional traffic from WFPs during peak hours, network may become even more congested, increasing the burden on an ISP. Moreover, a higher traffic load may also negatively affect the connection quality of users at the WFPs' end and degrade network performance. From a commercial perspective, while an WFP's users may gain Internet access and the WFP that charge users may receive some monetary gain, the ISP may have to incur higher cost to support the additional traffic without additional financial benefits.

B. Free Service, Flat-rate, and Dynamic Pricing

To decide an appropriate pricing mechanism for providing additional bandwidth, we first explore the tradeoffs between free service, flat rate, and dynamic pricing. Free Internet access is commonly employed by WFPs to entice customers to boost their business sales. For example, a coffee shop offers free Wi-Fi to attract more customers to come and linger, in order to achieve higher sales per customer. A WFP has very little incentive to provide additional bandwidth because it does not gain additional benefit from users enjoying good service, especially when good service has little direct impact on increasing the sale of their primary product/service. Similarly, flat-rate pricing strategy for Internet access also provides little incentive for a WFP to offer additional bandwidth. For example, a hotel guest may pay a daily flat rate ($10 per day) during his/her stay, and providing better service does not increase the hotel's revenue because the guest still pays the same rate regardless of the quality of the connection. By the same token, there is little incentive for an ISP to offer additional bandwidth if a WFP pays a flat rate to the ISP. As a result, without the ISP's support for providing additional bandwidth, the WFP may not be able to offer additional bandwidth even if it desires to.

Dynamic pricing is a usage-based pricing strategy where users pay according to the amount of bandwidth they use. This pricing model may provide more incentive to a WFP to offer additional bandwidth because users must pay more for a better connection quality, which leads to higher earning for the WFP. This pricing strategy can also be used to avoid congestion when there is more demand than available bandwidth in network. In such a situation, a WFP increases the price to reduce traffic load. Therefore, dynamic pricing not only offers more opportunities for higher revenue, it also gives a WFP better control over traffic. Similar arguments apply to an ISP on the employment of dynamic pricing to provide additional bandwidth to its WFP's users.

C. Cooperative Versus Non-Cooperative Strategies

In order to investigate what kind of sharing mechanism may provide incentive for an ISP to support its WFPs through additional bandwidth, we explore both *cooperative* and *non-cooperative* strategies.

Assumption 1. Price charged by a WFP to users, denoted by λ, is no less than the price charged by its ISP, denoted by g.

In a *non-cooperative* setting, an ISP determines and charges its WFPs with a price g. Then, a WFP sells bandwidth to users at price λ and pays the ISP at price g, where $g \leq \lambda$. Here, the WFP gets to keep the difference between λ and g. In all circumstances, the ISP will not know the WFP's final sale price λ to users. This scenario may not be favorable to the ISP when the WFP earns significantly higher revenue, and the ISP may want a higher share in the revenue. The non-cooperative setting may not be favorable to users either because the ISP may not gain any additional financial benefit from allocating more resources for better service. As a result, users may experience poor performance during peak time even after paying a high price to their WFP. Thus, non-cooperative model is not supportive to user demand for additional bandwidth.

In a *cooperative* setting, an ISP decides the minimum selling price g to its WFP and the WFP sets the final sale price λ to end users, and the ISP and the WFP share the total revenue received at price λ. In this setting, the ISP is informed of the final sale price λ to the users, and its portion of the revenue corresponds to the final price. Because it knows the final sale price to users, the ISP is able to monitor and assess the actual demand and the value of the service. Based on this reasoning, we propose in this paper a revenue sharing mechanism based on Shapley value [6]. This mechanism is desirable because it exhibits several fairness properties that ensure revenue sharing is proportional to each party's contribution to the total revenue.

D. Related Work

In [1], the authors propose a pricing strategy based on *online mechanism design* (OMD) to provide Wi-Fi service to Starbucks' customers. Their pricing strategy is designed on the basis of users dynamically arriving at and leaving the coffee shop in a period of time, and considers that users make certain decisions based on certain outcomes as time progresses. For instance, a customer may decide to leave the shop after he/she has finished his/her drink, or to stay longer for more drinks. In addition, their pricing strategy also requires users to reveal their true valuation on the Internet access and their arrival time at the coffee shop. Our proposal, on the other hand, does not require users to reveal their valuation of the service to WFP, and Wi-Fi price is determined according to network traffic and is available after users start transmitting data. Our approach provides more flexibility and the price can be updated dynamically in real time setting. Furthermore, the role of ISP is not incorporated in their design. Our model, on the other hand, allows ISP to influence WFP's pricing, especially when ISP is experiencing high traffic demand.

Revenue sharing between ISPs utilizing Shapley value has been studied in numerous literatures. For example, [8] has studied Shapley value to model ISPs' routing and interconnection decision. Similarly, [9] explores the design of profit sharing mechanism using Shapley value that allows the revenue to be decided "fairly" among participating ISPs. In [10], the authors examine the bilateral prices that can achieve the Shapley-value solution in ISP peering. These literatures primarily focus on revenue sharing between ISPs that are at par or almost equal. In other words, multiple ISPs are partnering and collaborating at equal or almost equal level to deliver massive amount of data from content providers (like Google, CNN, ESPM, etc) to a very large pool of users. Moreover, ISP's infrastructure is very complex and expensive which increases the complexity of the collaboration between ISPs. Our paper is mainly concerned with revenue sharing between ISP and WFP, the latter a customer of ISP, where WFP relies on and pays ISP for Internet access. Also, WFP's network infrastructure is made up of only consumer level routers, and it serves a much smaller pool of users. Thus, equal partnership between ISP and WFP is not possible in this setup. In [11], Shapley value is also incorporated in the revenue sharing mechanism between cellular network providers and wireless data plan subscribers (reseller) who resell their unused bandwidth to provide ad hoc Internet access for monetary rewards. The revenue sharing mechanism in [11] is targeted at low usage subscriber resellers, such that resellers' portion is only sufficient to offset their monthly subscription fee. There is however no restriction on the amount of revenue that can be generated by WFP and to be shared with ISP. [5] proposes auction based secondary market for ad-hoc Internet access, where the provider keeps the difference between the bids from users and the actual price.

Other than Shapley value based models, [12] proposes asymmetric Nash Bargaining Solution revenue sharing model between different types of ISPs. In this study, Stackelberg game, a *non-cooperative* game model, is considered in their pricing scheme, where a group of ISPs decide their prices according to the prices being offered in the market by other ISPs. However, Shapley value based revenue sharing model is is a *cooperative* based game theory [6]. In our discussion later, we show that the cooperative model is more favorable for ISP, and yields higher revenue for both ISP and WFP. The authors of [13] propose a revenue sharing mechanism between *global* WFPs (*Skype*) and *local* WFPs (coffee shops, hotels, etc.) and the mechanism to incentivize local WFP to support Skype. In this setup, Skype users completely rely on WFP to provide the additional bandwidth. Our model, in contrast, requires collaboration between WFP and ISP to provide additional bandwidth. Furthermore, our proposed revenue sharing model divides the revenue according to the contribution of the participating parties.

III. REVENUE SHARING

In this section, we introduce a revenue sharing model to address how the revenue gained from selling bandwidth to users is shared between a WFP and an ISP, by using revenue sharing mechanism based on Shapley value [6]. Figure 1 illustrates how revenue R is shared: ϕ_w apportioned to WFP w and ϕ_i to ISP i, depending on their relative contribution.

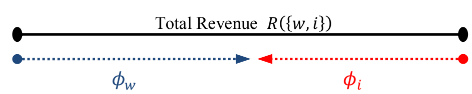

Fig. 1. Shapley value based revenue sharing.

A. Desirable Sharing Properties

We next list a set of properties that our revenue sharing mechanism should satisfy. Let $R(.)$ be the revenue function and variable ϕ denote a vector of *Shapley value*. Let $R(\{w, i\})$ denote the total revenue received from providing network service by WFP w which subscribes for bandwidth from ISP i. The Shapley value has the following desirable properties [16,17]:

Property 1 (efficiency): $\phi_w + \phi_i = R(\{w, i\})$.

The efficiency property requires that the total shared revenue equals the revenue received by providing service. In other words, the mechanism does not contribute or receive extra revenue.

Property 2 (symmetry): If $R(\mathbb{Z} \cup i) = R(\mathbb{Z} \cup w)$ (where $\mathbb{Z} = \emptyset$), then $\phi_w = \phi_i$.

The symmetry property requires that when an ISP and a WFP each renders the same contribution, both should receive the same portion of the revenue.

Property 3 (dummy player): If w is a dummy WFP, $R(w \cup i) = R(i)$ and $\phi_w = 0$.

This property assures that when WFP w does not contribute, then it receives zero share. Since WFP w relies on an ISP's infrastructure to sell bandwidth to users, the ISP always has a contribution in supporting the network service, but not necessarily WFP w.

Property 4 (fairness): For WFP w and ISP i, the portion of revenue share is proportional to their respective marginal contributions to the total revenue gained from the sale. This property addresses the fairness of revenue sharing between any pair of $\langle w, \mathrm{i}\rangle$.

Property 5 (additivity): Lets separate a transaction into two parts, such that $T = T_1 + T_2$. For any two transaction T_1 and T_2, $\phi_i(N, T_1 + T_2) = \phi_i(N, T_1) + \phi_i(N, T_2)$, where $N = \{w, i\}$ and $(N, T_1 + T_2)$ is defined by $(T_1 + T_2)S = T_1(S) + T_2(S)$ for every coalition of S.

The additivity property addresses the process of getting the Shapley value. The premise is that the outcome from transaction $(N, T_1 + T_2)$ should be equal to the addition of two different transactions of (N, T_1) and (N, T_2). To illustrate the idea, imagine a WFP receives revenue according to transaction (N, T_1) on the first day and (N, T_2) on the second day. Assume that T_1 and T_2 are independent. Then, the WFP's share from both days should be the summation of the revenue received from both transactions. In other words, this property guarantees that if the revenue of the service provided by the WFP is additive, then the distributed revenue is the sum of the revenue generated for providing the service.

B. Revenue Sharing Model

In this subsection, we propose a revenue sharing scheme between an ISP and a WFP and its implementation.

Definition 1. *Shapley value* ϕ is defined by

$$\phi_j = \frac{1}{|N|!} \sum_{\pi \in \Pi} \Delta_i\big(R, Z(\pi, i)\big), \qquad \forall j \in N, \tag{1}$$

where $N = \{w, i\}$, $Z \subseteq \mathrm{N}$, and

$$\Delta_j\big(R, Z(\pi, j)\big) = R(\, Z \cup \{j\} - R(Z)\,), \tag{2}$$

where $j \in N$. Remark: Given (N, R), consider a permutation on π on the set N. Members of set N appear to "collect" their revenues according to the ordering all possible permutation π. For each member in N, let Z_π^j be the set of members preceding member j, where $Z_\pi^j \subseteq N$. The marginal contribution of member j according to all possible permutation π is $\Delta_j\big(R, Z(\pi, j)\big) = R\left(Z_\pi^j \cup \{j\} - R(Z_\pi^j)\right)$. Here, the Shapley value can be interpreted as the expected marginal contribution $\Delta_j(R, Z)$, where Z is preceding j in an uniformly distributed random ordering. Since in this model $|N| = 2$, that is $N = \{w, i\}$, the Shapley value for w and i can be obtained by the following approach:

$$\phi_i = \frac{1}{2} R(\{i\}) + \frac{1}{2}\big(R(\{w, i\}) - R(\{w\})\big), \tag{3}$$

$$\phi_w = \frac{1}{2} R(\{w\}) + \frac{1}{2}\,\big(R(\{w, i\}) - R(\{i\})\big), \tag{4}$$

where $R(\{w, i\}) = \phi_i + \phi_w$. Here, ISP i will keep the revenue of ϕ_i amount, while WFP w will receive ϕ_w of the total revenue $R(\{w, i\})$, as illustrated in Figure 1.

Total revenue $R(\{w, i\})$ earned at the WFP's end is defined as follows.

$$R(\{w, i\}) = \sum_{s \in w} x_s\, \lambda_s^f\ . \tag{5}$$

Here x_s and λ_s^f denote bandwidth allocation and the final price charged by WFP w to user s respectively. Therefore,

$$R(\{w, i\}) = \sum_{s \in w} x_s\, \lambda_s^f \ = \ \phi_w + \phi_i\,,$$

where the term $s \in w$ denotes that user s receives Internet service provided by WFP w. Additionally, in Shapley value methodology, $R(\{w\})$ and $R(\{i\})$ are the contributions of the Wi-Fi provider and the ISP to revenue $R(\{w, i\})$ respectively, but they also can be interpreted as the revenue that they will gain if they do not collaborate. Here, the ISP revenue $R(\{i\})$ is determined by solving the following equation

$$R(\{i\}) = \sum_{s \epsilon w} x_s\, g_s\,, \tag{6}$$

where g_s denotes the price determined by the ISP to provide service to user s. Revenue $R(\{i\})$ can be interpreted as the cost charged by the ISP to the WFP at price g_s for providing service to user s. Equation (6) indicates that the ISP charges different prices to different users. However, since the WFP relies on the ISP to provide the network access, when $R(\{w\}) = 0$, then $R(\{i\}) = R(\{w, i\})$, which means ISP keeps the entire revenue of $R(\{w, i\})$. Thus,

$$R(\{i\}) = \begin{cases} \phi_i\,, & R(\{w\}) > 0 \\ \displaystyle\sum_{s \in w} x_s\, \lambda_s^f\,, & R(\{w\}) = 0. \end{cases} \tag{7}$$

Next, given that the ISP provides Internet access to the WFP, the WFP's contribution $R(\{w\})$ to $R(\{w, i\})$ is determined as follows.

$$R(\{w\}) = \frac{\sum_{s\,\epsilon\, w} \big(\lambda_s^f - g_s\big)x_s}{\max(log(g_w), \beta)}\,, \tag{8}$$

where $g_w = \sum_{s \epsilon w} g_s$ and $\beta \geq 1$ is a positive constant to guarantee the denominator is greater than 1. To assure that apportion of the share is allocated to the WFP, the denomination factor in (8) is concave and flattened as g_w increases. Moreover, the gap between λ_s^f and g_s is considered in equation (8) to assure that the WFP receives more if λ_s^f increases. However, without Internet access provided by the ISP, the contribution of the WFP is $R(\{w\}) = 0$. In our design of $R(\{w\})$, the WFP's marginal contribution is influenced by the cost incurred by the ISP to provide the access but the influence diminishes as the cost increases. A higher cost g_w of access results in a lower WFP contribution to the total revenue to a certain point, which may allow the WFP to achieve higher profit from the sales after the ISP's cost is covered. Next, we show that the property of our revenue sharing mechanism.

Theorem 1. The revenue sharing mechanism assures that an ISP's revenue portion at least covers the cost of providing service to its WFP, i.e.

$$\phi_i \geq \sum_{s \epsilon w} x_s\, g_s.$$

The Proof of theorem 1 is provided in the appendix. Additionally, by definition, a WFP's minimum revenue share is described by equation (8).

Naturally, a WFP and an ISP may want to maximize Shapley value in order to maximize their earnings. However, it is very difficult for WFPs to determine their maximum earnings because maximizing Shapley Value is coNP-hard [7]. The solution to Shapley Value maximization problem is discussed in [7].

The amount of revenue for an ISP and its WFP is clearly determined by the price charged to users. In line with the objective of our revenue sharing mechanism to incentivize an ISP and its WFP to offer additional bandwidth to users, we also introduce a dynamic pricing strategy in next section.

IV. PRICING MECHANISM

In this section, we propose an ISP's pricing strategy to its WFP and the WFP's pricing to user. Figure 2 illustrates the overview of a transaction: after a user s makes his/her request for connection, the ISP presents the WFP with the *minimum price* g_s. At the same time, the WFP computes its *price* λ_s, and determines the *final sale price* λ_s^f, formulated as

$$\lambda_s^f = max(\lambda_s , g_s + \rho), \quad (9)$$

where ρ denotes a constant minimum profit decided by the WFP, for $\rho \geq 0$. Then, the WFP presents price λ_s^f to user s and user s pays the WFP at price λ_s^f.

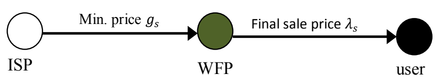

Fig. 2. Pricing mechanism.

Moreover, Figure 3 gives an overview of the pricing formulation in which the ISP decides the minimum sale price g_s to support user s and at the same time the WFP also decides the its own price λ_s to user s. Then, user s pays the service at price λ_s^f (the maximum between the two prices according to e.q (10)).

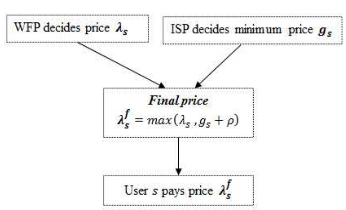

Fig. 3. Pricing Formulation.

The pricing mechanism also considers multiple users at any point of time. We begin by first addressing an WFP's price to user.

A. Wi-Fi Provider's Price

Let S_w denotes a set of users who access the Internet, through WFP w. For any $s \in S_w$, the objective of user s is to solve

$$max\, U(x_s , \lambda_s^f), \quad \text{for } x_s , \lambda_s^f \geq 0, \quad (10)$$

where x_s denotes the amount of data usage by user s and λ_s^t denotes the price to be paid by user s for Internet access at time t. The price is dynamically determined according to the level of demand for network service. The utility function of the user is defined as follows.

$$U(x_s , \lambda_s^f) = U_{bw}(x_s) + U_{cost}(x_s , \lambda_s^f),$$

where $U_{bw}(x_s)$ and $U_{cost}(x_s , \lambda_s^f)$ denote the utility of user s in terms of bandwidth consumption x_s and service cost, respectively. Considering that the WFP operates at frequency band B_s, the utility function of bandwidth usage is defined as follows

$$U_{bw}(x_s) = W_s \log\left(x_s\left(1 + \frac{P_s\,|c_s|^2}{\partial_s^2\, B_s}\right)\right),$$

where P_s is the transmission power of user s mobile device, c_s is the channel gain from WFP w to user s, and ∂_s^2 is the Gaussian noise variance for the channel between w and s [14]. In other words, $U_{bw}(x_s)$ is influenced by the channel quality and the amount of bandwidth. Additionally, $U_{bw}(x_s)$ follows the law of diminishing return. This is because more bandwidth does not always mean higher satisfaction and SNR measurement for wireless is concave [14].

Utility function $U_{cost}(x_s, \lambda_s^f)$ represents user satisfaction for monetary surplus when the cost paid for Internet access is less than the budget, which is defined as follows.

$$U_{cost}(x_s , \lambda_s^f) = 1 - \frac{x_s\, \lambda_s^f}{m_s},$$

where m_s denotes the budget that user s is willing to spend for bandwidth x_s. Note that $x_s\, \lambda_s^t$ can be interpreted as the price that user s must pay for the service. Thus, ideally, user's budget matches the price that he/she must pay for the service, such that $\frac{x_s \lambda_s^f}{m_s} = 1$ and hence $U_{cost}(x_s , \lambda_s^f) = 0$. Therefore, given final price λ_s^f, user s utilizes m_s to influence the amount of bandwidth x_s allocated to him/her.

The objective of WFP w is to maximize its own revenue without exceeding its monthly bandwidth capacity. The maximization problem is expressed as follows.

$$max \sum_{s\,\epsilon\, w} x_s\, \lambda_s^f\, , \quad (11)$$

$$s.t. \sum_{s\,\epsilon\, w} x_s \leq C_w\, ,$$

$$over\; x_s \geq 0,\;\; \forall s \in w,$$

where $s\,\epsilon\, w$ denotes user s receives from w, capacity C_w is amount of the WFP's bandwidth capacity. Considering the respective objectives of users and the WFP at the system level, the problem can be formulated into network utility maximization (NUM) [2].

$$max \sum_{s\,\epsilon\, w} U(x_s, \lambda_s^f) \quad (12)$$

$$s.t. \sum_{s\,\epsilon\, w} x_s \leq C_w\, ,$$

$$\text{over } x_s \geq 0,\;\; \forall s \in w.$$

In this setup, the solution for problem (12) also solves problem (9) and (11). The Lagrangian optimization problem is formulated as

$$L(\bar{x}_s, \bar{\lambda}_s) = \sum\nolimits_{s\epsilon w} U(x_s, \lambda_s^f) - \sum_{s\epsilon w} x_s \lambda_s + \sum_{s\epsilon w} \lambda_s C_w,$$

where $L(.)$ is the Lagrangian form and λ_s is known as the Lagrangian multiplier, which is often interpreted as the link price, and $\bar{x}_s$ is a vector of x_s, for $\forall s \,\epsilon\, w$, and $\bar{\lambda}_s$ is a vector of λ_s . The common solution to NUM problem is the subgradient based method [3]. Typically, the dual problem D to the primal problem of (12) is constructed as follows $\min D(\bar{\lambda}_s)$, $s.t \;\; \bar{\lambda}_s \geq 0$, where the dual function

$$D(\bar{\lambda}_s) = \max_{0 \leq \bar{x}_s \leq x^{max}} L(\bar{x}_s, \bar{\lambda}_s).$$

To solve $D(\bar{\lambda}_s)$, user s maximizes over x_s given λ_s. That is

$$x_s = \underset{0 \leq x_s \leq x_s^{max}}{\arg\max} \left(U(x_s, \lambda_s)\right). \tag{13}$$

However , since e.q. (10) assures the minimum price charged to users. Thus, e.q. (13) can be expressed as follows.

$$x_s = \underset{0 \leq x_s \leq x_s^{max}}{\arg\max} \left(U(x_s, \lambda_s^f)\right)$$

Next, $L(\bar{x}_s, \bar{\lambda}_s)$ is minimized with subgradient projection method in an iterative solution given by

$$\lambda_s = \left[\lambda_s - \sigma^t \left(C_s - \sum_{s\epsilon w} x_s\right)\right]^+, \tag{14}$$

where $C_s - \sum_{s\epsilon S} x_s$ is a subgradient of $D(\lambda_s)$ and σ^t denote the step size to control the tradeoff between a convergence guarantee and the convergence speed, such that

$$\sigma^t \rightarrow 0, as\; t \rightarrow \infty \; and \; \sum_{t=1}^{\infty} \sigma^t = \infty. \tag{15}$$

Next, after solving λ_s, then we solve for λ_s^f with (10). Notice that in (10), $g_s + \rho$ serves the minimum price charged to user s, such that $\lambda_s^f \geq g_s + \rho$. Generally, the subgradient-based solution relies on feedback loop mechanism. That is, the user determines the transmission rate according to the price set by the WFP by solving (13) and the price is adjusted according to the traffic load by solving (14). It is repeated until it converges to an optimal solution. Price λ_s is also an indication of the demand for service. However, before determining the final sale price, the WFP must consider the minimum price charged by the ISP as described in (10). It is because the WFP depends on the ISP's infrastructure to provide the service. The discussion on the minimum price is addressed in the next section.

Proposition 1: If the step size σ in (15) satisfies (14), then the subgradient-based algorithm converges to the optimal solution of problem (12). [15] ■

B. Minimum Price by ISP

Here, we address how an ISP determines the minimum price for bandwidth. Consider a network managed by an ISP with a set of links L, and a set of link capacities C over the links in L. Given a utility function $U_s(x_s, \lambda_s^f)$ of data user s with bandwidth usage of x_s and traffic generated by users in S, the maximization problem can be formulated as follows.

$$max \sum\nolimits_{s\in S} U(x_s, \lambda_s^f) \tag{16}$$

$$s.t. \; \sum_{s\epsilon S} x_s \leq C_l \,,$$

$$\text{over } x_s \geq 0.$$

Here, users in set S include all users who get Internet service from WFPs that supported by the ISP. To solve problem (16), users in S solve eq. (13) and the ISP determines the minimum price to sell on each link l by solving

$$g_l = \left[g_l - \sigma^t \left(\left(C_l - \sum_{s\epsilon S} x_s\right) - \sum_{s\in l} \sum_{s\epsilon w} x_s\right)\right]^+. \tag{17}$$

Here, $s \in l$ denotes user s who transmits data through link l. The total *minimum price* to sell to user s is

$$g_s = \sum_{s\in l} g_l \,, \forall s, s \,\epsilon\, w.$$

Proposition 2: If the step size σ in (15) satisfies (17), then the solution converges to the optimal solution of (16). [15] ■

Since the speed of convergence is determined by step size σ, the running time required to obtain convergence also depends on the value of σ. Higher value of σ increases the speed of convergence but it may have the risk that the algorithm does not converge. Similarly, lower value of σ decreases the convergence speed but increases the convergence guarantee.

V. Simulation and Discussion

The objective of our simulation is to understand the behavior of revenue sharing between an ISP and a WFP using the Shapley Value model. More specifically, we investigate how the difference between the final sale price paid by users and the minimum price set by ISP influences revenue sharing, and whether the outcome is favorable to ISP or WFP. In the simulation setup, we have a WFP subscribing Internet access from an ISP to provide Wi-Fi to users. In this setup, the WFP provides to users bandwidth of 10 MB/sec and the initial minimum price charged by the ISP is 10 units currency and the total minimum profit desired by the WFP is 5 units currency. Thus, the initial price charged to users is 15 units currency. This set-up applies to the two scenarios we are investigating: first, congestion occurs at the ISP's network, and second, there is a high demand at the WSP's end but low traffic load in the ISP's network. In each scenario, either the ISP's or the WFP's price is raised incrementally by one unit up to 300 increments.

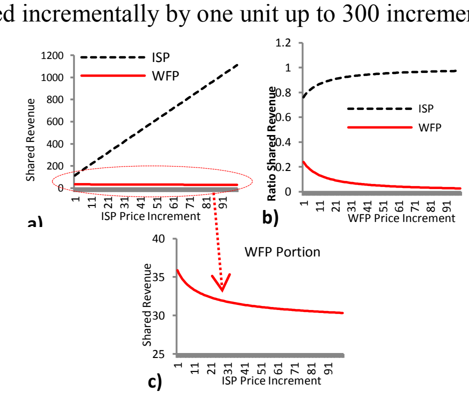

Fig. 4. Revenue sharing when ISP increases the price. (a) The revenue division between ISP and WFP in unit currency (e.g., dollar). (b) The revenue division between ISP and WFP in percentage. (c) The revenue that is apportioned to WFP in unit currency.

First scenario. During the peak hours of an ISP's network, the ISP increases price to reduce the amount of demand. In this scenario, there are ten users getting Internet access from a Wi-Fi provider, where each user receives equal amount of bandwidth allocation. Figure 4(a) and 4(b) illustrate that our proposed revenue sharing mechanism favors the ISP during its peak hours, at the same time the model also assures that the WFP receives some portion of the revenue. In other words, the majority of the revenue is allocated to the ISP during peak hours. The y-axis of figure 4(a) depicts the portion allocated to the ISP and the WFP in unit currency and the y-axis in Figure 4(b) is the percentage of revenue apportioned to the WFP and the ISP, totaling to 100%. Therefore, there is little incentive for the WFP to provide additional bandwidth during peak hours. However, as Figure 4(c) demonstrates, regardless how much ISP charges, WFP always receives some portion of the revenue as WFP's portion converges to a value even when ISP's price continues to grow. In essence, the outcome from this scenario implies that during peak hours ISP receives most of the share of the revenue, regardless how much WFP charges users

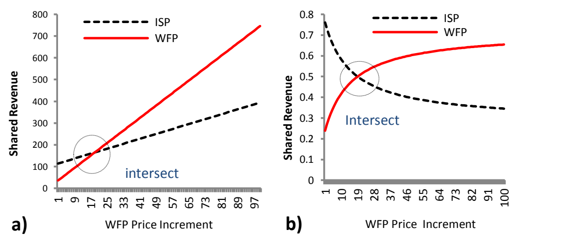

Fig. 5. Revenue sharing when WFP increases the price and the bargaining power shift between ISP and WFP. (a) The revenue division between ISP and WFP in unit currency (e.g., dollar). (b) The revenue division between ISP and WFP in percentage.

Second scenario. The WFP receives a high level of demand from users, causing the WFP to increase its price. However, there is low traffic at the ISP's end, where the minimum price decided by the ISP is much lower than the WFP's final price. In this second scenario, there are initially ten users getting Internet access but eventually the number of users is increased by five in each iteration. Moreover, the amount of bandwidth is divided equally among users, which results in a smaller allocation to each user as the number of users increases. Figures 5(a) and 5(b) illustrate the division of revenue between the WFP and the ISP as the WFP's price is incrementally raised by 1 unit currency up to 300 increments. Figure 5(a) depicts the division of revenue in unit currency and Figure 5(b) in percentage value.

Figure 5(a) shows that in this scenario both the ISP and the WFP receive a higher revenue from the high demand. The higher revenue received by the ISP should provide an incentive for the ISP to provide additional bandwidth. From the first to the 22nd price increment, as shown in figure 5(a), ISP receives a higher share of the revenue. Up to this point the revenue is still relatively low. This can be interpreted as because the ISP provides and manages the infrastructure, a higher portion of the revenue is allocated to the ISP to cover the cost of providing the access and managing the traffic from the WFP.

At 22nd price increment, the two lines intersect (Figure 4) at price incremental at 22. The intersection can be interpreted as when the bargaining power is balanced and revenue is equally shared between the ISP and the WFP. However, as the WFP generates higher revenue and the traffic at the ISP's end remains low, the bargaining power progressively shifts toward the WFP. As the WFP price increases, its bargaining power also increases because of its higher "contribution" to the transaction. The WFP's portion of revenue eventually converges to a region of 70% of the total revenue (figure 5(b)), and the ISP's percentage share converges to 30%. Generally, the convergence to a specific value shows asymptotically that there is a predictable region of revenue sharing. Importantly, this convergence also provides the upper celling of the portion allocated to the WFP, i.e. 70%, and the bottom limit of the portion allocated to the ISP, i.e. 30%. The existence of these upper bounds for revenue sharing can become the basis for both the WFP and the ISP to evaluate and negotiate the risk and gain in such trade agreement for mutual benefits.

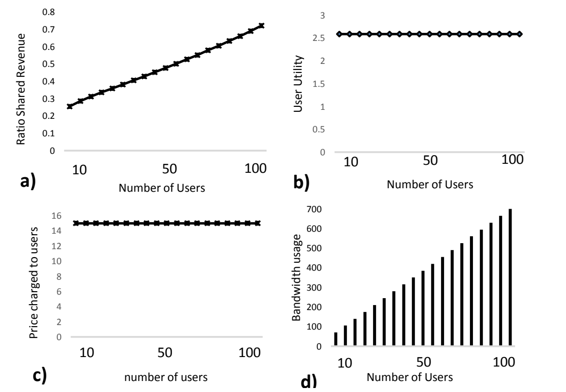

Fig. 6. Revenue sharing when bandwidth demand is *low* with up to 100 users. (a) Ratio shared revenue of WFP over ISP. (b) Average user utility. (c) The price that users pay for the service. (d) Amount of bandwidth sold to users.

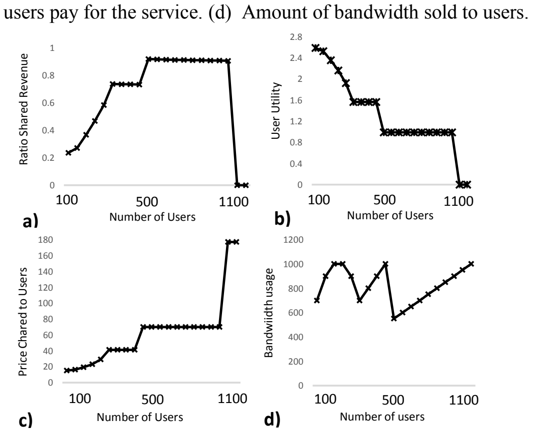

Fig. 7. Revenue sharing when bandwidth demand is *high* with up 1100 users. (a) Ratio shared revenue of WFP over ISP. (b) Average user utility. (c) The price that users pay for the service. (d) Amount of bandwidth sold to users.

Third scenario: we investigate the correlation between revenue sharing, pricing, users' utility, and bandwidth usage. The simulation setup includes an WFP providing 1000MB/sec to users with initial minimum price charged by an ISP of 10 units currency, minimum profit desired by the WFP is 5 units currency, users maximum willingness to pay is 100 units currency, and user utility and price are measured and determined using eq. (10) and (14) respectively. In this scenario, we consider two case studies: (i) When the WFP experiences *lower* and (ii) *higher* demand for bandwidth. **Case** (i)**:** There are 10 to 100 users acquiring service from WFP.

Figure 6(a) illustrates that shared revenue ratio of $\frac{WFP\ shared\ revenue}{ISP\ shared\ revenue}$ increases as the number of users increases, which confirms our previous simulation results. Figure 6(b) and Figure 6(c) demonstrate that users' average utility and price are stable when there is sufficient bandwidth for users. That is when the total bandwidth usage of 100 users described in Figure 6(d) is only 700 MB/sec $<$ 1000 MB/sec. **Case** (***ii***): There are 100 to 1100 users subscribing from the WFP. The steep incline depicted in Figure 7(a) shows that the WFP rapidly achieves higher shared revenue as demands for bandwidth increase, and the WFP quickly attains near equal share as the ISP. In addition, the behavior of shared revenue illustrated in Figure 7(c) is also a reflection of price movement caused by the WFP hiking the price up when the total demands exceeds the capacity limit. This leads to the bandwidth fluctuation illustrated in Figure 7(d) as a result from users adapting their demand for bandwidth when the price increases. In addition, Figure 7(a-c) reach the plateau (or flat) whenever bandwidth usage falls below the capacity limit, but change when demands go over the capacity limit. Moreover, Figure 7(c) also shows that user utility decreases as the price increases, because users obtain less bandwidth for higher price. Then, user utility in Figure 7(d) drops to zero when the price in Figure 7(b) peaks at 172 units currency, which results in unaffordable service leading to zero transaction. This also means no revenue for both the WFP and the ISP, as described in Figure 7(a). In conclusion, a WFP achieves higher shared revenue when there is high demand for bandwidth until the price becomes unaffordable, but this is also at the cost of lower user utility. On the other hand, we also demonstrate that a WFP also can obtain higher shared revenue and allow an ISP to gain higher revenue while achieving high user utility, when the bandwidth usage nears the limit capacity while keeping the price stable. Similar outcome is expected when an ISP price is increased beyond users' affordability except higher shared revenue will apportioned to an ISP relative to a WFP.

VI. Conclusion

In this paper, we propose a Shapley value based revenue sharing scheme and a NUM based dynamic pricing strategy to provide incentives for ISP and WFP to offer additional bandwidth to their users. We explored the revenue sharing model in two alternative scenarios, cooperative and non-cooperative, and discovered that the cooperative model offers better incentives for ISP. In the cooperative revenue sharing setting, ISP will be aware of WFP's final sale price to users, which gives ISP better understanding of the market and more control over pricing. Importantly, our revenue sharing model is able to address critical concerns such as traffic management. Additionally, our model also captures the conditions in which one of the parties (ISP or WFP) receives a higher portion of the revenue. For instance, the sharing model apportions a larger share of the revenue to ISP when WFP generates lower revenue, but favors WFP when it brings higher revenue. When WFP contributes a significant portion of the revenue, the division of revenue eventually converges to a stable value, which gives larger share to WFP. We also demonstrate it is possible for a WFP and an ISP to achieve higher revenue while attain high user utility. In our future work, we will investigate whether the economic interplay and negotiation between ISP, WFPs, and users can reach an equilibrium.

Appendix

Proof Theorem 1.

By definition defined in e.q. (10) that $g_s \le \lambda_s^f$, $\lambda_s^f - g_s \ge 0$. Thus, by comparing $\lambda_s^f - g_s$ and $R(\{w\})$ in e.q. (8), we have the following equality.

$$\sum_{s\,\epsilon\,w} (\lambda_s^f - g_s) \ge \frac{\sum_{s\,\epsilon\,w} (\lambda_s^f - g_s)}{\max(log(g_w), \beta)}. \quad (18)$$

Observe that in the equality above, as the $g_w = \sum_{s\epsilon w} g_s$ increases, the right side of the equality decreases quicker than the left side. Next, the equality (18) can be derived further as follows.

$$\sum_{s\,\epsilon\,w} \lambda_s^f - \sum_{s\,\epsilon\,w} g_s \ge \frac{\sum_{s\,\epsilon\,w} (\lambda_s^f - g_s)}{\max(log(g_w), \beta)},$$

Which is also

$$\sum_{s\epsilon w} g_s \le \sum_{s\epsilon w} \lambda_s^f - \frac{\sum_{s\epsilon w} (\lambda_s^f - g_s)}{\max(log(g_w), \beta)}.$$

By considering bandwidth x_s allocated for every user s that receives service from WFP w, equality (18) also implies

$$\sum_{s\epsilon w} (x_s\, g_s) \le \sum_{s\epsilon w} (x_s\, \lambda_s^f) - \frac{\sum_{s\epsilon w} (\lambda_s^f - g_s)\, x_s}{\max(log(g_w), \beta)}. \quad (19)$$

Now, consider combining (3) with (5), (6), and (8), the revenue shared apportion to ISP is

$$\phi_i = \frac{1}{2} \sum_{s\epsilon w} (x_s\, g_s) + \frac{1}{2}\left(\sum_{s\epsilon w} (x_s\, \lambda_s^f) - \frac{\sum_{s\epsilon w} (\lambda_s^f - g_s)\, x_s}{2\ \max(log(g_w), \beta)} \right). \quad (20)$$

Next, we substitute (19) to (20) and then we have

$$\phi_i \ge \frac{1}{2} \sum_{s\epsilon w} (x_s\, g_s) + \frac{1}{2} \sum_{s\epsilon w} (x_s\, g_s) = \sum_{s\epsilon w} (x_s\, g_s).$$

Thus, $\phi_i \ge \sum_{s\epsilon w} (x_s\, g_s)$, which is the revenue shared apportion to ISP covers the minimum cost. ■

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