New Tight Upper Bounds on the Capacity for General Deterministic Dissemination in Wireless Ad Hoc Networks

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***Abstract*—In this paper, we study capacity scaling laws of the deterministic dissemination (DD) in random wireless networks under the generalized physical model (GphyM). This is truly not a new topic. Our motivation to readdress this issue is two-fold: Firstly, we aim to propose a more general result to unify the network capacity for general homogeneous random models by investigating the impacts of different parameters of the system on the network capacity. Secondly, we target to close the open gaps between the upper and the lower bounds on the network capacity in the literature. We derive the general upper bounds on the capacity for the arbitrary case of (λ, n_d, n_s) by introducing the Poisson Boolean model of continuum percolation, where λ, n_d, and n_s are the general node density, the number of destinations for each session, and the number of sessions, respectively. We prove that the derived upper bounds are tight according to the existing general lower bounds constructed in the literature.**

***Index Terms*—Network Capacity, Scaling Laws, Deterministic Dissemination, Random Wireless Networks, Percolation Theory**

I. Introduction

This work falls within the scope of the issue of capacity scaling laws for wireless networks, initiated by Gupta and Kumar [1], *i.e.*, the scaling of network performance in the limit when the network gets large, [2]. The main advantage of studying scaling laws is to highlight qualitative and architectural properties of the system without considering too many details [1], [2]. The network capacity depends directly on the type of traffic sessions of interest. Generally, the traffic sessions in wireless networks can be classified into two broad types: *data dissemination*, where a session has only one source, and *data gathering*, where a session intends to transmit data from its multiple sources to a relatively small number of destinations; on the other hand, according to the property of destination selection schemes, they can also be divided into the following two types: *deterministic session*, where the selection of destination(s) is determined beforehand, and *opportunistic session*, where the destination(s) are/is opportunistically chosen during the transmitting procedure. Based on those classifications, the typical session patterns can be located as shown in Table. I.

In this work, we focus on dissemination sessions that can be usually represented by a triple dimensional vector (n, n_c, n_d) with $1 \leq n_d \leq n_c \leq n-1$. These parameters are defined by the following: The node set of network, say $\mathcal{V} := \mathcal{V}(n)$, comprises n nodes; the cardinality of source set $\mathcal{S} \subseteq \mathcal{V}$ is $|\mathcal{S}| = n_s$; during the process of dissemination with source $v_i \in \mathcal{S}$, n_c nodes are randomly chosen to compose a *candidate set*, denoted by $\mathcal{C}_i$, and the session is completed when data are transmitted to a subset $\mathcal{D}_i \subseteq \mathcal{C}_i$, called *destination set*, where $|\mathcal{D}_i| = n_d \leq n_c$. Obviously, when $n_d \equiv n_c$, the dissemination is specified into a *deterministic dissemination*, *i.e.*, the so-called *general multicast session*. Please see the illustration in Fig.1.

The purpose of this paper is to investigate the capacity of wireless networks where $n_s : (1, n]$[1] general multicast sessions, denoted by (n, n_d, n_d) with n_d:$[1, n]$, run simultaneously. In the research of *networking-theoretic* capacity scaling laws [2], the unicast and broadcast sessions can be usually regarded as two special cases of general multicast sessions according to the number of destinations for each session. Usually, any proposed multicast capacity could be specialized into the unicast and broadcast capacities by letting $n_d = 1$ and $n_d = n$, respectively. This principle often applies in the literatures [3]–[8].

Most of the existing results differ from each other due to the diversity of adopted analytical models and assumptions. Besides session patterns introduced above, there are two typical models in terms of scaling patterns that are adopted in the literature: *random extended network* (REN), where the node density is fixed to a constant [5], [7], [9]–[11], and *random dense network* (RDN), where the node density increases linearly with the number of nodes [1], [4], [12]–[14]. In [13], Shakkottai *et al.* derived the multicast capacity of RDN for a specific case that $n_s = n^{\epsilon}$ and $n_s \cdot n_d = \Theta(n)$, where $\epsilon \in (0, 1]$. They showed that such per-session multicast capacity under the protocol model is at most of $O(\frac{1}{\sqrt{n_s \log n}})$. To achieve the upper bound, they proposed a simple and novel routing architecture, called the *multicast comb*, to transfer

[1]We use the term $f(n) : [\phi_1(n), \phi_2(n)]$ to represent $f(n) = \Omega(\phi_1(n))$ and $f(n) = O(\phi_2(n))$; and use $f(n) : (\phi_1(n), \phi_2(n))$ to represent $f(n) = \omega(\phi_1(n))$ and $f(n) = o(\phi_2(n))$.

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multicast data in the network. A more general result, in terms of n_s and n_d, was proposed by Li ***et al.*** in [3]. They showed that when $n_s = \Omega(\log n_d \cdot \sqrt{n \log n/n_d})$, the per-session multicast capacity for RDN under the protocol model is of $\Theta(\frac{1}{n_s}\sqrt{\frac{n}{n_d \log n}})$ if $n_d = O(\frac{n}{\log n})$, and is of $\Theta(1/n_s)$ if $n_d = \Omega(n/\log n)$. After that, Keshavarz-Haddad ***et al.*** [4] derived the multicast capacity for RDN under the ***generalized physical model*** [15] by designing new multicast schemes and computing the upper bounds. A ***gap*** remains open between the upper and the lower bounds in the regime $n_d : [n/(\log n)^3, n/\log n]$. For multicast capacity of REN under the generalized physical model, Li ***et al.*** [5] derived a lower bound as $\Omega(\frac{\sqrt{n}}{n_s\sqrt{n_d}})$ for the case that $n_s = \Omega(n^{1/2+\epsilon})$ and $n_d = O(n/(\log n)^{2\alpha+6})$. Recently, Wang ***et al.*** [7] devised the specific multicast schemes and derived the multicast throughput for all cases n_s:$(1, n]$ and n_d:$[1, n]$. Under the assumption that $n_s = \Theta(n)$, their lower bounds are specialized into those in Equation (4). They also derived an upper bound for the case that $n_s = \Theta(n)$, as in Equation (5). An obvious ***gap*** exists between the upper and the lower bounds in the regime n_d:$[n/(\log n)^{\alpha+1}, n/\log n]$. To the best of our knowledge, this is the latest results on general multicast capacity for static REN without considering the impacts of node mobility [6], [16] or advanced physical communication technology [17]–[19].

Closing the remaining ***gaps*** is one of the motivations of this paper.

Both REN and RDN are extreme cases for a random network consisting of n nodes in terms of the node density λ. So the characterization of two particular models does not suffice to develop a comprehensive understanding of wireless networks, although they are representative models to some extent, [2]. Hence, in this paper, we comprehensively consider the network with a general node density $\lambda : [1, n]$, rather than only the cases $\lambda = 1$ (REN) and $\lambda = n$ (RDN), which can offer complete and deep insights about the scaling laws for wireless networks. ***Unearthing*** the ***nature*** of general scaling is another motivation of this work.

In conclusion, we aim to examine the capacity scaling laws of general wireless networks, where the generality lies in three aspects: (1) a general node density, λ:$[1, n]$; (2) a general number of receivers, $n_d : [1, n]$; (3) a general number of sessions, n_s:$(1, n]$. For such general multicast capacity of general wireless networks, we have computed the lower bounds under the generalized physical model in [20]. More specifically, we build routing backbones of two levels: ***highways*** and ***arterial roads***. Furthermore, arterial roads (ARs) have two subclasses, ***i.e.***, ***ordinary arterial roads*** (O-ARs) and ***parallel arterial roads*** (P-ARs). Note that the highways are the same as those in [4], [5], [7], [9], but the ARs are different from the ***second-class highways*** (SHs) in [7]. Recall that in the SH system of [7], there are two types of SHs: ***odd*** SHs and ***even*** SHs. The bottleneck of the whole routing could happen in the switching phase between the odd and even SHs. There is no such a bottleneck in the current AR system, which can

TABLE I: Typical Session Patterns

	Deterministic Session	Opportunistic Session
Dissemination /Single-Source	Unicast Broadcast Multicast	Anycast $\cdots$ Manycast
Gathering /Multiple-Sources	Data Collection $\cdots$ ConvergeCast (Many-to-One)	Undefined (to the best of our knowledge)

improve the multicast throughput for some regimes of n_s and n_d. Based on the highways, O-ARs and P-ARs, we design four routing schemes. By exploiting the theory of ***maximum occupancy***, we derive the optimal multicast throughput and scheme according to different ranges of λ, n_d, and n_s.

Major contributions of this paper can be summarized as follows:

$\triangleright$ For deriving the upper bounds on multicast capacity, we introduce the Poisson Boolean model of continuum percolation [21] (not Poisson bond percolation model [9]), which, to the best of our knowledge, is not used in previous studies on upper bounds of network capacity. Based on the argument of ***giant cluster*** (component) in the Poisson boolean percolation model, we can divide the communications under any multicast routing scheme into two parts, ***i.e.***, communications inside and outside the giant component. Obviously, the network throughput must be determined by the bottleneck of two parts. We give a general formula to compute upper bounds on the capacity.

$\triangleright$ For the case that $n_s = \Theta(n)$ and $\lambda = n$ (or $\lambda = 1$), ***i.e.***, RDN and REN, due to the limitations of adopted analytical methods, the previous works [4], [7] have not derived the tight bounds on multicast capacity under the generalized physical model. By applying our general results to these special cases, we close those gaps.

The rest of the paper is organized as follows. The system model is formulated in Section II. In Section III, we present and discuss the main results. We make preparations for the analysis of network capacity in Section IV, and discuss the features of network topology under the feasible routing schemes in Section V. Based on these results, we finally derive the upper bounds on the capacity in Section VI. We draw some conclusions in Section VII.

II. System Model

A. Random Scaling Model

We construct a random network with node density λ, denoted by $\mathcal{N}(\lambda, n)$, by placing wireless nodes randomly into a square region $\mathcal{R}(\lambda, n) = [0, \sqrt{A}]^2$ according to a Poisson point process with density λ, where $A = n/\lambda$. When λ is set to be 1 (or n), our model corresponds to ***random extended network*** (REN) (or ***random dense network*** (RDN)), [2]. According to Chebyshev's inequality, we get that the number of nodes in $\mathcal{A}(a^2)$ is within $((1-\epsilon)n, (1+\epsilon)n)$ with high probability, where $\epsilon > 0$ is an arbitrarily small constant, [22]. To simplify the description, we assume that the number of nodes is exactly n, without changing our results in the

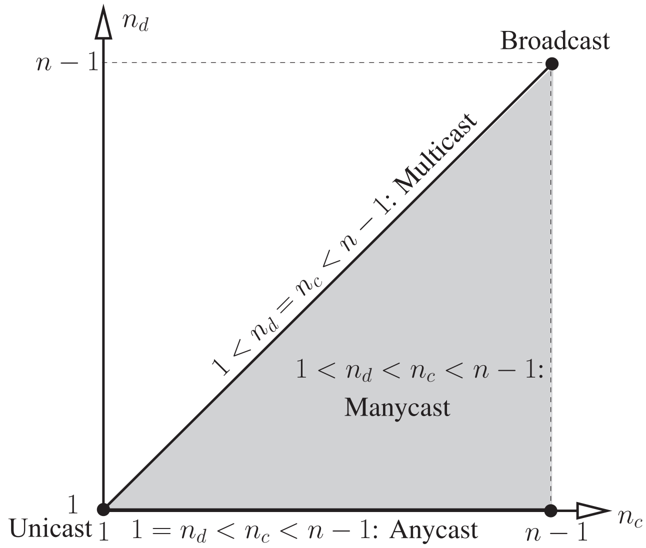

Fig. 1: General Dissemination Sessions. Here, m is the size of the specified group for a source, where $1 \le m \le n-1$; d is the number of intended destinations of the source, where $1 \le d \le m$.

sense of order, [7], [9], [10]. We are mainly concerned with the events that occur inside these squares with high probability (w.h.p.); that is, with probability approaching one as $n \to \infty$.

B. System Model

In wireless networks, there are two broad types of session patterns: ***information dissemination*** [23] and ***information gathering*** [22]. The former is the interest of this paper. Generally, dissemination sessions can be further divided into two categories: ***deterministic dissemination***, in which the destination(s) of a message is (are) determined when it is generated at a source, such as unicast [24], broadcast [25], and multicast [26], and ***opportunistic dissemination***, such as anycast [27], [28], and manycast [29] sessions, in which the destination(s) of a message is (are) opportunistically chosen and both the paths to the group member(s) and the destination(s) can change dynamically according to the network condition, such as the node movement situation.

In this work, we focus on the general multicast sessions, including unicast, broadcast and multicast sessions. We adopt a similar construction procedure to the one in [7]. To generate the k-th ($1 \le k \le n_s$) multicast session, with source $v_{\mathcal{S},k} \in \mathcal{S}$, denoted by $\mathcal{M}_{\mathcal{S},k}$, n_d points $p_{\mathcal{S},k_i}$ ($1 \le i \le n_d$, and $1 \le n_d \le n-1$) are randomly and independently chosen from the deployment region $\mathcal{R}(\lambda, n)$. Denote the set of these n_d points by $\tilde{\mathcal{P}}_{\mathcal{S},k} = \{p_{\mathcal{S},k_1}, p_{\mathcal{S},k_2}, \cdots, p_{\mathcal{S},k_{n_d}}\}$. Let $v_{\mathcal{S},k_i}$ be the nearest ad hoc node from $p_{\mathcal{S},k_i}$ (ties are broken randomly). In $\mathcal{M}_{\mathcal{S},k}$, the node $v_{\mathcal{S},k}$, serving as a source, intends to deliver data to n_d destinations $\mathcal{D}_{\mathcal{S},k} = \{v_{\mathcal{S},k_1}, v_{\mathcal{S},k_2}, \cdots, v_{\mathcal{S},k_{n_d}}\}$ at an arbitrary data rate $\lambda_{\mathcal{S},k}$. Let $\mathcal{U}_{\mathcal{S},k} = \{v_{\mathcal{S},k}\} \cup \mathcal{D}_{\mathcal{S},k}$ be the ***spanning set*** of nodes for the multicast session $\mathcal{M}_{\mathcal{S},k}$. Please see the illustration in Fig.2.

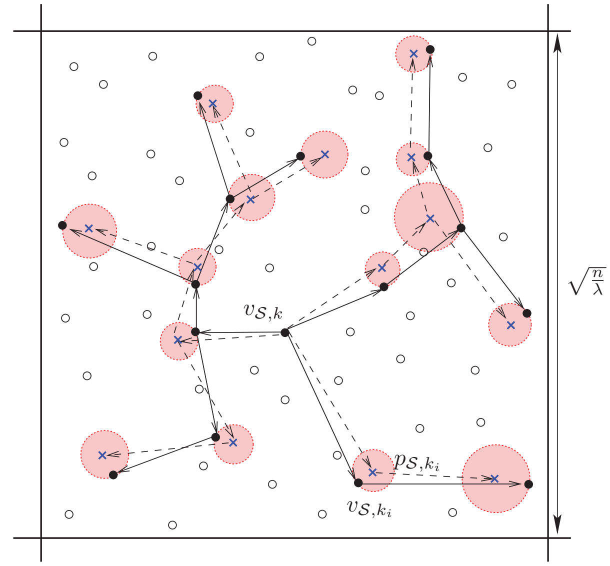

Fig. 2: Multicast session $\mathcal{M}_{\mathcal{S},k}$, [7]. The tree consisting of solid lines represents the Euclidean minimum spanning tree (EMST) over $\mathcal{U}_{\mathcal{S},k} = \{v_{\mathcal{S},k_i} \mid 0 \le i \le n_d\}$, denoted by $\mathrm{EMST}(\mathcal{U}_{\mathcal{S},k})$, where $v_{\mathcal{S},k_0}$ is $v_{\mathcal{S},k}$. The tree consisting of dashed lines represents an Euclidean spanning tree (EST) over $\mathcal{P}_{\mathcal{S},k} = \{p_{\mathcal{S},k_i} \mid 0 \le i \le n_d\}$, denoted by $\mathrm{EST}_0(\mathcal{P}_{\mathcal{S},k})$, where $p_{\mathcal{S},k_0}$ is $v_{\mathcal{S},k}$, and for any $0 \le i,j \le n_d$, link $(p_{\mathcal{S},k_i} \to p_{\mathcal{S},k_j}) \in \mathrm{EST}_0(\mathcal{P}_{\mathcal{S},k})$ if and only if link $(v_{\mathcal{S},k_i} \to v_{\mathcal{S},k_j}) \in \mathrm{EMST}(\mathcal{U}_{\mathcal{S},k})$.

C. Communication Model

Generally, there are three types of communication (interference) models: the *protocol model* [1], *physical model* [1] and *generalized physical model* [15] (along with the name Gaussian Channel model, [5]). We adopt the generalized physical model since it is more realistic than the other two [5], [9], [14], [15].

Let $\mathcal{K}_t$ denote a *scheduling set* of links in which all links can be scheduled simultaneously in time slot t.

Definition 1. *Under the generalized physical model, when a scheduling set $\mathcal{K}_t$ is scheduled, the rate of a link $<u,v> \in \mathcal{K}_t$ is achieved at*

$$R_{u,v;t} = B \times \mathbf{1} \cdot \{<u,v> \in \mathcal{K}_t\} \times \log(1 + \mathrm{SINR}_{u,v;t}), \quad (1)$$

where $\mathrm{SINR}_{u,v;t} = \frac{P \cdot \ell(|\mathbf{x}_u - \mathbf{x}_v|)}{N_0 + \sum_{<i,j> \in \mathcal{K}_t / <u,v>} P \cdot \ell(\mathbf{x}_i - \mathbf{x}_v|)}$*; $\mathbf{x}_u$ denotes the position of node u, $|\mathbf{x}_u - \mathbf{x}_v|$ represents the Euclidean distance between node u and node v; $\ell(\cdot)$ denotes the power attenuation function that is assumed to depend only on the distance between the transmitter and the receiver [1], [5], [9], [30]; $\ell(|\cdot|) := |\cdot|^{-\alpha}$ for dense scaling networks, and $\ell(|\cdot|) := \min\{1, |\cdot|^{-\alpha}\}$ for extended scaling networks [9].*

III. Main Results

We mainly derive the upper bounds on the general multicast capacity of random ad hoc networks.

A. General Upper Bounds

Theorem 1. *The multicast capacity for random network $\mathcal{N}(\lambda, n)$ is at most*

$$\overline{\Lambda}(\lambda, n) = \max_{l_c:\mathcal{L}_c}\left\{\min\left\{\frac{\min\{1, l_c^{-\alpha}\}}{\mathbf{L}(n_s, \frac{\sqrt{n}}{l_c\sqrt{n_d\lambda}})}, \frac{\min\{1, (\frac{\lambda}{\log n})^{\frac{\alpha}{2}}\}}{\mathbf{L}(n_s, \frac{n\cdot\sqrt{\lambda}\cdot l_c}{n_d\cdot\sqrt{\log n}})}\right\}\right\},$$

where $\mathcal{L}_c = [1/\sqrt{\lambda}, \sqrt{\log n/\lambda}]$, and $\mathbf{L}(m, n)$ is defined in Table II.

B. Tight Capacity Bounds

In [20], the general lower bounds have been provided by designing some strategies.

Lemma 1 ([20]). *The general multicast throughput for random network $\mathcal{N}(\lambda, n)$ can be achieved of*

$$\underline{\Lambda}(\lambda, n) = \max\{\Lambda_{\mathrm{o}}(\lambda, n), \Lambda_{\mathrm{p}}(\lambda, n), \Lambda_{\mathrm{o\&h}}(\lambda, n), \Lambda_{\mathrm{p\&h}}(\lambda, n)\},$$

where $\Lambda_{\mathrm{o}}(\lambda, n), \Lambda_{\mathrm{p}}(\lambda, n), \Lambda_{\mathrm{o\&h}}(\lambda, n), \Lambda_{\mathrm{p\&h}}(\lambda, n)$ are defined in Table II.

We specialize the general results from Theorem 1 and Lemma 1 to the cases that $\lambda = n$ and $\lambda = 1$, corresponding to the RDN and REN. Following a common assumption in most existing works, *i.e.*, $n_s = \Theta(n)$, we show that for both RDN and REN our results give the first tight bounds on multicast capacity over the whole regime $n_d : [1, n]$.

1) Random Dense Networks: In Theorem 1, $\overline{\Lambda}(n, n)$, *i.e.*, the upper bound on the capacity achieves its maximum value by choosing $l_c = \Theta(\frac{1}{\sqrt{n}})$ when $n_d = O(n/(\log n)^2)$; and also achieves its maximum value by choosing $l_c = \Theta(\sqrt{\log n}/\sqrt{n})$ when $n_d = \Omega(n/(\log n)^2)$. Specifically, the multicast capacity is *at most* of order

$$\begin{cases} \Theta(\frac{1}{\sqrt{n_d n}}) & \text{when} \quad n_d : [1, \frac{n}{(\log n)^3}] \\ \Theta(\frac{1}{n_d(\log n)^{\frac{3}{2}}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^3}, \frac{n}{(\log n)^2}] \\ \Theta(\frac{1}{\sqrt{n n_d \log n}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ \Theta(\frac{1}{n}) & \text{when} \quad n_d : [\frac{n}{\log n}, n] \end{cases} \tag{2}$$

This result is *exciting*, because the multicast throughput as in Equation (2) had been proven to be achievable by Keshavarz-Haddad *et al.* in [4]. Moreover, they derived an upper bound as

$$\begin{cases} O(\frac{1}{\sqrt{n_d n}}) & \text{when} \quad n_d : [1, \frac{n}{(\log n)^2}] \\ O(\frac{1}{n_d\cdot\log n}) & \text{when} \quad n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ O(\frac{1}{n}) & \text{when} \quad n_d : [\frac{n}{\log n}, n] \end{cases} \tag{3}$$

It is clear that there is a gap between the upper and the lower bounds in the regime $n_d : (\frac{n}{(\log n)^3}, \frac{n}{\log n})$. In this work, we *close* this gap. Moreover, by Lemma 1, this optimal throughput in Equation (2) can also be achieved by using our schemes $\mathbb{M}_{\mathrm{o}}$ cooperatively and $\mathbb{M}_{\mathrm{o\&h}}$ that are defined in Table A.1 in Appendix A of our technical report [31].

2) Random Extended Networks: In Theorem 1, $\overline{\Lambda}(1, n)$ achieves its maximum value by letting $l_c = \Theta(1)$ when $n_d = O(n/(\log n)^2)$; and achieves its maximum value by letting $l_c = \Theta(\sqrt{\log n})$ when $n_d = \Omega(n/(\log n)^2)$. Specifically, the multicast capacity is *at most* of order

$$\begin{cases} \Theta(\frac{1}{\sqrt{n_d n}}) & \text{when} \quad n_d : [1, \frac{n}{(\log n)^{\alpha+1}}] \\ \Theta(\frac{1}{n_d(\log n)^{\frac{\alpha+1}{2}}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^{\alpha+1}}, \frac{n}{(\log n)^2}] \\ \Theta(\frac{1}{\sqrt{n n_d}\cdot(\log n)^{\frac{\alpha-1}{2}}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ \Theta(\frac{1}{n_d(\log n)^{\frac{\alpha}{2}}}) & \text{when} \quad n_d : [\frac{n}{\log n}, n] \end{cases} \tag{4}$$

Such multicast throughput had been achieved by the schemes in [7]. The upper bounds were proposed as:

$$\begin{cases} O(\frac{1}{\sqrt{n_d n}}) & \text{when} \quad n_d : [1, \frac{n}{(\log n)^{\alpha}}] \\ O(\frac{1}{n_d(\log n)^{\frac{\alpha}{2}}}) & \text{when} \quad n_d : [\frac{n}{(\log n)^{\alpha}}, n] \end{cases} \tag{5}$$

That is, we close the gap between the upper and the lower bounds in the regime $n_d : [\frac{n}{(\log n)^{\alpha+1}}, \frac{n}{\log n}]$. In addition, by Lemma 1, this optimal throughput in Equation (4) can be equally achieved by using our schemes $\mathbb{M}_{\mathrm{p}}$ and $\mathbb{M}_{\mathrm{p\&h}}$ cooperatively that are defined in Table A.1 in Appendix A of our technical report [31].

IV. Technical Preparations

A. Maximum Occupancy

We use the results in *maximum occupancy theory* to derive the lower bounds of the multicast throughput. Now we introduce the following result from [32], [33] and [34].

Lemma 2. *Let $\mathbf{L}(m, n)$ be the random variable that counts the maximum number of balls in any bin, if we throw m balls independently and uniformly at random into n bins. Then, it holds that* w.h.p..

$$\mathbf{L}(m, n) = \begin{cases} \Theta\left(\frac{\log n}{\log\frac{n}{m}}\right) & \textit{when} \quad m : [1, \frac{n}{\mathrm{polylog}(n)}) \\ \Theta\left(\frac{\log n}{\log\frac{n\log n}{m}}\right) & \textit{when} \quad m : [\frac{n}{\mathrm{polylog}(n)}, n\log n) \\ \Theta\left(\frac{m}{n}\right) & \textit{when} \quad m = \Omega(n\log n) \end{cases}$$

B. Network Throughput by Occupancy Theory

We give a technical lemma as a basic argument of the analysis of network capacity.

Lemma 3. *Given a multicast scheme $\mathbb{M}$, for any link initiating from a node u, say uv, if it can sustain a rate of $\mathbf{R}(\lambda, n)$, and any multicast session shares the bandwidth of link uv with the probability of $\mathbf{p}$, then the throughput along link uv is of order $\Theta(\Lambda(\lambda, n))$, where $\Lambda(\lambda, n) = \frac{\mathbf{R}(\lambda,n)}{\mathbf{L}(n_s,1/\mathbf{p})}$.*

C. The Tail of Poisson Trials

Lemma 4 ([35]). *Let X_1, X_2, $\cdots$, X_n be independent Poisson trials such that, for $1 \le i \le n$, $\Pr[X_i = 1] = p_i$, where $0 < p_i < 1$. Then, for $X = \sum_{i=1}^n X_i$, $\mu = \mathrm{E}(X) = \sum_{i=1}^n p_i$, and any $\delta > 0$,*

$$\Pr[X > (1+\delta)\mu] < \left[\frac{e^\delta}{(1+\delta)^{1+\delta}}\right]^\mu.$$

D. Euclidean Spanning Tree

Lemma 5 ([7]). *If X_i, $1 \leq i \leq \infty$, are uniformly distributed on $[0, a]^d$, for a set $\mathcal{U}(n) = \{X_1, X_2, \cdots, X_n\}$, denote its Euclidean minimum spanning tree (EMST) by $\mathrm{EMST}(\mathcal{U}(n))$. Under such deployment model, build $K(n)$ sets, denoted by $\mathcal{U}_1(n), \mathcal{U}_2(n), \cdots, \mathcal{U}_{K(n)}(n)$, it holds that*

$$\Pr\left[\lim_{n\to\infty} \frac{\sum_{k=1}^{K(n)} \|\mathrm{EMST}(\mathcal{U}_k(n))\|}{K(n) \cdot a \cdot n^{1-\frac{1}{d}}} = \nu(d)\right] = 1. \quad (6)$$

This lemma can be straightforwardly proven according to Theorem 2 of [36]. Please see the detailed proof of Lemma D in the appendices of [7].

For any n_s general multicast sessions constructed by the method in Section II-B, by a similar procedure to Lemma 7 of [7], we have,

Lemma 6. *For all multicast sessions $\mathcal{M}_{\mathcal{S},k}$ ($1 \leq k \leq n_s$), it holds that for $n_d = o(\frac{n}{\log n})$,*

$$\sum\nolimits_{k=1}^{n_s} \|\mathrm{EMST}(\mathcal{D}_{\mathcal{S},k})\| = \Omega(n_s \cdot \sqrt{n_d \cdot n}),$$

where $\mathrm{EMST}(\mathcal{D}_{\mathcal{S},k})$ denotes the Euclidean minimum spanning tree (EMST) over the destination set $\mathcal{D}_{\mathcal{S},k}$.

Note that the session construction in this work is different from that in [7], and Lemma 6 only gives a result on $\sum_{k=1}^{n_s} \|\mathrm{EMST}(\mathcal{D}_{\mathcal{S},k})\|$ instead of $\sum_{k=1}^{n_s} \|\mathrm{EMST}(\mathcal{M}_{\mathcal{S},k})\|$, where $\mathrm{EMST}(\mathcal{M}_{\mathcal{S},k})$ denotes the Euclidean minimum spanning tree (EMST) over the spanning set $\mathcal{U}_{\mathcal{S},k}$. Since it holds that $\|\mathrm{EMST}(\mathcal{M}_{\mathcal{S},k})\| \geq \|\mathrm{EMST}(\mathcal{D}_{\mathcal{S},k})\|$, we can obtain the following corollary.

Corollary 1. *For all multicast sessions $\mathcal{M}_{\mathcal{S},k}$ ($1 \leq k \leq n_s$), it holds that for $n_d = o(\frac{n}{\log n})$,*

$$\sum\nolimits_{k=1}^{n_s} \|\mathrm{EMST}(\mathcal{M}_{\mathcal{S},k})\| = \Omega(n_s \cdot \sqrt{n_d \cdot n}).$$

V. Network Topology under Feasible Routings

We introduce the Poisson Boolean percolation model to make preparations for computing the upper bounds on the general multicast capacity.

A. Poisson Boolean Percolation Model

In a 2-dimensional Poisson Boolean model $\mathbb{B}(\lambda, r)$ [21], nodes are distributed in $\mathbb{R}^2$ according to a p.p.p of intensity λ. Each node is associated with a closed disk of radius $r/2$. Two disks are *directly connected* if they overlap. Two disks are *connected* if there exist a sequence of directly connected disks between them. Define a *cluster* as a set of disks in which any two disks are connected. Denote the set of all clusters by $\mathscr{C}(\lambda, r)$. Let $|\mathcal{C}_i|$ denote the number of disks in a cluster $\mathcal{C}_i \in \mathscr{C}(\lambda, r)$. We can associate $\mathbb{B}(\lambda, r)$ with a graph $\mathcal{G}(\lambda, r)$, called an *associated graph*, by associating a vertex with each node in $\mathbb{B}(\lambda, r)$ and an edge with each direct connection in $\mathbb{B}(\lambda, r)$. Two models $\mathbb{B}(\lambda, r)$ and $\mathbb{B}(\lambda_0, r_0)$ lead to the same associated graph, namely $\mathcal{G}(\lambda, r) = \mathcal{G}(\lambda_0, r_0)$ if $\lambda_0 {r_0}^2 = \lambda r^2$. Then, the graph properties of $\mathbb{B}(\lambda, r)$ only depend on the parameter λr^2, [37]. Let $\mathcal{C}$ denote the cluster containing the given node, the percolation probability is thus defined as $\Pr_{\lambda,r}[|\mathcal{C}| = \infty]$. We call γ_c the *critical percolation threshold* of Poisson Boolean model in $\mathbb{R}^2$ when

$$\gamma_c = \sup\{\gamma := \lambda\pi r^2 \mid \Pr_{\lambda,r}[|\mathcal{C}| = \infty] = 0\}.$$

The exact value of γ_c is still open. The analytical results show that it is within the range $(0.7698\pi,\ 3.372\pi)$ [21], [38]. In terms of the value of $\gamma = \lambda\pi r^2$, the *subcritical phase* and *supercritical phase* can be defined, which correspond to the cases when $\gamma < \gamma_c$ and $\gamma > \gamma_c$, respectively. The following lemma will be used in our analysis.

Lemma 7 ([21], [39]). *For a Poisson Boolean model $\mathbb{B}(\lambda, r)$ in $\mathbb{R}^2$, there exists a value γ_c in a square region $\mathcal{R}(\lambda, n) = [0, \sqrt{n/\lambda}]^2$, as $n \to \infty$:*

- *if $\gamma = \lambda\pi r^2 < \gamma_c$, i.e., in the subcritical phase [21], it holds that*
  $$\Pr[\sup\{|\mathcal{C}_i| \mid \mathcal{C}_i \in \mathscr{C}(\lambda, r)\} = O(\log n)] = 1;$$
- *if $\gamma = \lambda\pi r^2 > \gamma_c$, i.e., in the supercritical phase [21], there exists,* w.h.p., exactly *one* giant cluster *(*giant component*)* $\mathcal{C}_i \in \mathscr{C}(\lambda, r)$ *of size* $|\mathcal{C}_i| = \Theta(n)$.

B. Distance to Giant Component

Connectivity is a necessary condition for a feasible routing scheme. From [40], [41], the connectivity of a routing scheme for homogeneous random networks $\mathcal{N}(\lambda, n)$ can be ensured when the maximum link length can reach $\Omega(\sqrt{\log n/\lambda})$. More specifically, by a geometric extension, we can obtain the following lemma based on Theorem 3.2 of [40].

Lemma 8. *In Poisson Boolean model $\mathcal{B}(\lambda, r)$, with*

$$\pi \cdot \lambda \cdot r^2 = \log n + \varsigma(n),$$

all disks with radius r are connected *with probability 1 as $n \to \infty$ if and only if $\varsigma(n) \to \infty$.*

From Lemma 8, we limit the nontrivial range of r in $[\mathfrak{p}_c/\sqrt{\lambda}, \sqrt{\log n/\lambda}]$, *i.e.*, $r\ :\ [1/\sqrt{\lambda}, \sqrt{\log n/\lambda}]$. According to Lemma 7, in the Poisson Boolean model $\mathcal{B}(\lambda, r)$, there exists exactly one giant component, denoted by $\mathcal{C}(\lambda, r)$, with $|\mathcal{C}(\lambda, r)| = \Theta(n)$. Note that we take no account of the specific values of the involved constants, since they have no impact on the order of our final results.

In Poisson Boolean model $\mathcal{B}(\lambda, r)$, for any node outside the giant cluster $\mathcal{C}(\lambda, r)$, say an *exterior node* $u \notin \mathcal{C}(\lambda, r)$, we define the distance between u and the giant component by

$$\bar{l}_c(u) = \min\nolimits_{v\in\mathcal{C}(\lambda,r)} |uv|.$$

Furthermore, we define the largest distance between exterior nodes and $\mathcal{C}(\lambda, r)$ as

$$\bar{l}_c^{\mathrm{M}}\left[\mathcal{C}\left(\lambda, r\right)\right] := \max\nolimits_{u\in\mathcal{V}(n)-\mathcal{C}(\lambda,r)} \bar{l}_c(u),$$

where $\mathcal{V}(n)$ denotes the set of all nodes in $\mathcal{N}(\lambda, n)$. Please see the illustration in Fig.3.

From Lemma 8, there is no node outside $\mathcal{C}(\lambda, r)$ when

TABLE II: Defined Functions and Parameters.

Functions	Definitions
$\mathbf{L}(m,n)$	$\begin{cases} \Theta\left(\frac{\log n}{\log \frac{n}{m}}\right) & \text{when} \quad m:[1, \frac{n}{\mathrm{polylog}(n)}) \\ \Theta\left(\frac{\log n}{\log \frac{n \log n}{m}}\right) & \text{when} \quad m:[\frac{n}{\mathrm{polylog}(n)}, n\log n) \\ \Theta\left(\frac{m}{n}\right) & \text{when} \quad m = \Omega(n \log n) \end{cases}$
$\mathbf{R}_{\mathrm{O-AR}}(\lambda,n)$	$\begin{cases} \Theta(\frac{\lambda^{\frac{\alpha}{2}}}{(\log n)^{\frac{\alpha}{2}}}) & \text{when} \quad \lambda:[1,\log n] \\ \Theta(1) & \text{when} \quad \lambda:[\log n, n] \end{cases}$
$\mathbf{R}_{\mathrm{P-AR}}(\lambda,n)$	$\begin{cases} \Theta(\frac{\lambda^{\frac{\alpha}{2}}}{(\log n)^{\frac{\alpha}{2}}}) & \text{when} \quad \lambda:[1,(\log n)^{1-\frac{2}{\alpha}}] \\ \Theta(\frac{1}{\log n}) & \text{when} \quad \lambda:[(\log n)^{1-\frac{2}{\alpha}}, n] \end{cases}$
$\mathbf{p}_{\mathrm{o}}$	$\begin{cases} \Theta(\sqrt{\frac{n_d \log n}{n}}) & \text{when } n_d = O(\frac{n}{\log n}) \\ \Theta(1) & \text{when } n_d = \Omega(\frac{n}{\log n}) \end{cases}$
$\mathbf{p}_{\mathrm{p}}$	$\begin{cases} \Theta(\frac{\sqrt{n_d}}{\sqrt{n \log n}}) & \text{when } n_d:[1,\frac{n}{\log n}] \\ \Theta(\frac{n_d}{n}) & \text{when } n_d:[\frac{n}{\log n}, n] \end{cases}$
$\mathbf{p}_{\mathrm{oh,O-AR}}$	$\begin{cases} \Theta(\frac{n_d \cdot (\log n)^{3/2}}{n}) & \text{when } n_d:[1,\frac{n}{(\log n)^{3/2}}] \\ \Theta(1) & \text{when } n_d:[\frac{n}{(\log n)^{3/2}}, n] \end{cases}$
$\mathbf{p}_{\mathrm{oh,H}}$, $\mathbf{p}_{\mathrm{ph,H}}$	$\begin{cases} \Theta(\sqrt{\frac{n_d}{n}}) & \text{when } n_d:[1,\frac{n}{(\log n)^2}] \\ \Theta(\frac{n_d \log n}{n}) & \text{when } n_d:[\frac{n}{(\log n)^2}, \frac{n}{\log n}] \\ \Theta(1) & \text{when } n_d:[\frac{n}{(\log n)}, n] \end{cases}$
$\mathbf{p}_{\mathrm{ph,P-AR}}$	$\begin{cases} \Theta(\frac{n_d \cdot \sqrt{\log n}}{n}) & \text{when } n_d:[1,\frac{n}{\sqrt{\log n}}] \\ \Theta(1) & \text{when } n_d:[\frac{n}{\sqrt{\log n}}, n] \end{cases}$
$\Lambda_{\mathrm{o}}(\lambda,n)$	$\mathbf{R}_{\mathrm{O-AR}}(\lambda,n)/\mathbf{L}(n_s,\frac{1}{\mathbf{p}_{\mathrm{o}}})$
$\Lambda_{\mathrm{p}}(\lambda,n)$	$\mathbf{R}_{\mathrm{P-AR}}(\lambda,n)/\mathbf{L}(n_s,\frac{1}{\mathbf{p}_{\mathrm{p}}})$
$\Lambda_{\mathrm{o\&h}}(\lambda,n)$	$\min\left\{\frac{\mathbf{R}_{\mathrm{O-AR}}(\lambda,n)}{\mathbf{L}(n_s,\frac{1}{\mathbf{p}_{\mathrm{oh,O-AR}}})}, \frac{1}{\mathbf{L}(n_s,\frac{1}{\mathbf{p}_{\mathrm{oh,H}}})}\right\}$
$\Lambda_{\mathrm{p\&h}}(\lambda,n)$	$\min\left\{\frac{\mathbf{R}_{\mathrm{P-AR}}(\lambda,n)}{\mathbf{L}(n_s,\frac{1}{\mathbf{p}_{\mathrm{ph,P-AR}}})}, \frac{1}{\mathbf{L}(n_s,\frac{1}{\mathbf{p}_{\mathrm{ph,H}}})}\right\}$

$$\lambda \cdot r^2 = \tfrac{1}{\pi} \cdot (\log n + \varsigma(n)) \text{ if } \varsigma(n) \to \infty.$$

Then, we only consider the case that $\lambda \cdot r^2 = o(\log n)$, *i.e.*, $r = o(\sqrt{\log n/\lambda})$. It holds that

$$\bar{l}_c^{\mathrm{M}}\left[\mathcal{C}\left(\lambda, r\right)\right] > r, \text{ and } \bar{l}_c^{\mathrm{M}}\left[\mathcal{C}\left(\lambda, r\right)\right] = o(\sqrt{\log n/\lambda}).$$

Next, we give a useful result for computing the upper bounds on network capacity.

Lemma 9. *In Poisson Boolean model $\mathcal{B}(\lambda, r)$ with $r = o(\sqrt{\log n/\lambda})$, it holds that*

$$\lambda \cdot r \cdot \bar{l}_c^{\mathrm{M}} = \Omega(\log n), \text{w.h.p.}, \tag{7}$$

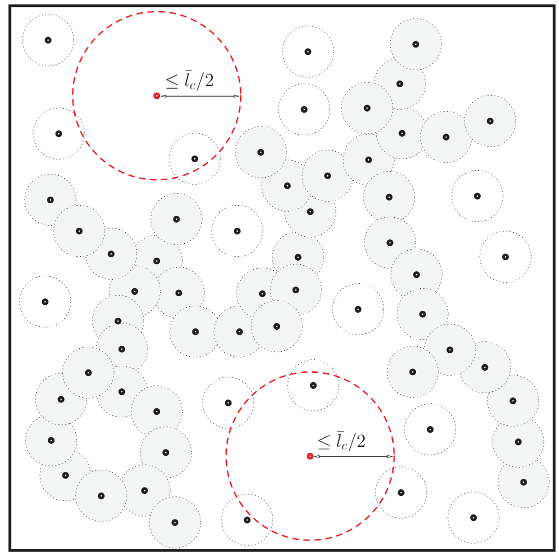

Fig. 3: Distance between an exterior node and the giant component.

where $\bar{l}_c^{\mathrm{M}} := \bar{l}_c^{\mathrm{M}}\left[\mathcal{C}\left(\lambda, r\right)\right]$ for the sake of succinctness.

Prior to proving Lemma 9, we get the following lemma based on Corollary 1 of [42] by a geometric scaling method.

Lemma 10 ([42], [43])**.** *For any exterior node, say $u \notin \mathcal{C}(\lambda, r)$, it holds that for any $x \in [0, \sqrt{n/\lambda}]$,*

$$\lim_{n\to\infty} \log \Pr\left[\bar{l}_c(u) > x\right] = -\lim_{n\to\infty} \varepsilon \cdot \lambda \cdot r \cdot x,$$

where $\varepsilon > 0$ is a constant.

Proof of Lemma 9. Firstly, we give a bound on the probability of event

$$\bar{E}(r) : \lambda \cdot r \cdot \bar{l}_c^{\mathrm{M}} = o(\log n),$$

which is a contradiction to Equation (7). For any $u \notin \mathcal{C}(\lambda, r)$, we define an event

$$\bar{E}_u(r)\text{: } \lambda \cdot r \cdot \bar{l}_c(u) = o(\log n).$$

Then, it follows that

$$\Pr\left[\bar{E}(r)\right] = \Pr\left[\bigwedge\nolimits_{u \notin \mathcal{C}(\lambda,r)} \bar{E}_u(r)\right] \leq \left(1 - \frac{\varepsilon_1}{o(n)}\right)^{\varepsilon_2 n},$$

where ε_1 and ε_2 are some constants. Hence, we have

$$\Pr\left[\bar{E}(r)\right] \to 0,$$

which proves this lemma. □

VI. Upper Bounds on General Multicast Capacity

For any routing scheme, denote the maximum length (in the sense of order) of the links by l_c. According to [1], [9], in the networking-theoretic scaling laws [7], under the premise of ensuring routing connectivity, long-distance communication is not preferable, since the interference generated would preclude too many nodes from communicating. The optimal strategy

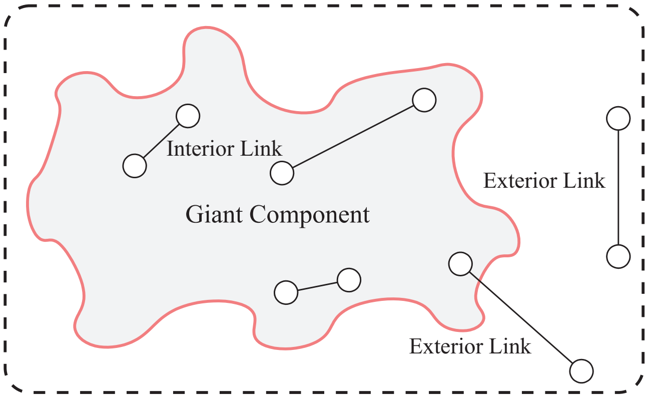

Fig. 4: Interior and Exterior Links.

is to confine to the nearest neighbor communication and maximize the number of simultaneous transmissions, *i.e.*, optimize the spatial reuse, [2], [44]. From [40], [41], the routing connectivity of any scheme for homogeneous random networks [45] can be ensured when the maximum link length is set to be $\Omega(\sqrt{\log n/\lambda})$. Then, we consider the range

$$l_c : [\mathfrak{p}_c/\sqrt{\lambda}, \sqrt{\log n/\lambda}], \textit{ i.e.}, \ l_c : [1/\sqrt{\lambda}, \sqrt{\log n/\lambda}].$$

From Lemma 7, in the Poisson Boolean model $\mathcal{B}(\lambda, l_c)$, there exists exactly one giant component, denoted by $\mathcal{C}(\lambda, l_c)$, with $|\mathcal{C}(\lambda, l_c)| = \Theta(n)$. Note that we take no account of the specific values of the constants, for they have no impact on the order of our final results.

Then, the links of any multicast scheme can be divided into two classes as follows: A link is called an *interior link*, if both endpoints are located in $\mathcal{C}(\lambda, l_c)$; and it is called an *exterior link*, otherwise. In the Poisson Boolean model $\mathcal{B}(\lambda, l_c)$, for any node outside the giant cluster $\mathcal{C}(\lambda, l_c)$, say $u \notin \mathcal{C}(\lambda, l_c)$, define the distance between u and the giant component as

$$\bar{l}_c(u) = \min\nolimits_{v \in \mathcal{C}(\lambda, l_c)} |uv|.$$

Furthermore, we define

$$\bar{l}_c^{\mathrm{M}} \left[\mathcal{C}\left(\lambda, l_c\right)\right] := \max\nolimits_{u \notin \mathcal{C}(\lambda, l_c/2)} \bar{l}_c(u).$$

Please see the illustration in Fig.3.

We derive the upper bounds on multicast capacity by considering two types of links comprehensively.

1) Inside a Giant Component: All links inside $\mathcal{C}(\lambda, l_c)$ have the length of $\Theta(l_c)$. The upper bound on capacity of these links can be computed as

$$\mathbf{R}_{l_c} = \min\left\{1, B\log\left(1 + \frac{l_c^{-\alpha}}{N_0}\right)\right\} = O(\min\{1, l_c^{-\alpha}\}).$$

Then, by combining with Lemma 3, we can obtain the following lemma.

Lemma 11. *For any multicast scheme with the parameter l_c, the multicast throughput along the links inside $\mathcal{C}(\lambda, l_c)$ is at most of order*

$$\Lambda_{l_c} = O\left(\frac{\min\{1, l_c^{-\alpha}\}}{\mathbf{L}\left(n_s, \frac{\sqrt{n}}{l_c\sqrt{n_d\lambda}}\right)}\right).$$

Proof. By Lemma 5, the length of any multicast tree is at least of order $\Omega(\sqrt{n_d n/\lambda})$. Then, for a given sender of any links inside the giant component, a multicast session passes through it with a probability of

$$\Omega\left(\min\left\{1, \frac{l_c\sqrt{n_d n/\lambda}}{n/\lambda}\right\}\right), \textit{ i.e.}, \ \Omega\left(\min\left\{1, \frac{l_c\sqrt{n_d\lambda}}{\sqrt{n}}\right\}\right).$$

By Lemma 3, the proof is completed. □

2) Outside a Giant Component: Based on Lemma 9, we have,

Lemma 12. *For any multicast scheme with l_c, the multicast throughput along the links between $\mathcal{C}(\lambda, l_c)$ and the nodes outside is at most of order*

$$\Lambda_{\bar{l}_c^{\mathrm{M}}} = O\left(\frac{\min\{1, (\frac{\lambda}{\log n})^{\alpha/2}\}}{\mathbf{L}(n_s, \frac{n\sqrt{\lambda}\cdot l_c}{n_d\cdot\sqrt{\log n}})}\right).$$

Proof. Since there must be a link outside the giant component with the length of $\sqrt{\log n/\lambda}$, the link capacity is bounded by

$$\begin{aligned} \mathbf{R}_{\bar{l}_c^{\mathrm{M}}} &= \min\left\{1, B\log\left(1 + \frac{(\sqrt{\log n/\lambda})^{-\alpha}}{N_0}\right)\right\} \\ &= O\left(\min\left\{1, \left(\frac{\lambda}{\log n}\right)^{\alpha/2}\right\}\right). \end{aligned}$$

From Lemma 9, $\bar{l}_c^{\mathrm{M}} = \Omega\left(\frac{\log n}{\lambda\cdot l_c}\right)$. It implies that $\bar{l}_c^{\mathrm{M}} = \Omega(\sqrt{\log n/\lambda})$ because $l_c : [1/\sqrt{\lambda}, \sqrt{\log n/\lambda}]$. The probability that a multicast session passes through such a link is of

$$\Omega\left(\min\left\{1, \frac{n_d\cdot\bar{l}_c^{\mathrm{M}}\bar{l}_c^{\mathrm{M}}\cdot\sqrt{\lambda}}{n\cdot\sqrt{\log n}}\right\}\right).$$

By Lemma 3, the proof is completed. □

By combining Lemma 11 and Lemma 12, we finally obtain Theorem 1.

VII. Conclusion and Discussion

We derive some new upper bounds on the capacity for general deterministic dissemination in random wireless networks with a general node density. When the general results are specialized to the well-known random dense and extended networks, we show that our results close the previous gaps between the upper and the lower bounds on the multicast capacity for both networks.

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